Stringham

WENTWORTH AND HILL'S

Examination Manuals

No. II.

ALGEBRA.

l.p.

Irving Stringham

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PREFACE.

THIS Manual consists of two parts: The first part contains one hundred and fifty examination papers, the questions for which have been selected mainly from the best English, French, and German collections of problems. These papers may be divided into three groups. The first fifty papers embrace the subjects of Elementary Algebra as far as Quadratic Equations; the next fifty papers also include Quadratic Equations and Radical Expressions; the last fifty papers extend over still more ground, including several topics usually regarded as belonging to Higher Algebra.

In each of these groups the earlier papers will be found somewhat easier than the later ones. The papers are intended to be *hour* papers, but if any of them are thought to be too long for one hour, the time may be increased or the length of the paper reduced by omitting one or more of the questions.

The second part of the Manual is a collection of recent papers actually set in various American and English institutions of learning.

There are two ways in which the Manual may be used.

First: To *test* the learner's knowledge in the usual way by means of an examination. For this purpose the class will come to the recitation-room provided with the Manual and blank books, and the teacher will simply designate by number the paper to be worked.

Secondly: To *exercise* the learner from day to day in the various rules and processes, to detect his weak points, and ascer-

tain where he most needs assistance. This may be done by assigning exercises to be worked in the class-room, or by assigning to each member of the class a paper with directions to hand in the solutions, neatly worked out, at a subsequent recitation.

The Manual will be found especially useful in reviewing the subject of Algebra, and in preparing for examinations.

Answers to the problems in the first one hundred and fifty papers, bound separately in paper covers, can be had by teachers *only*, on application to the publishers.

G. A. WENTWORTH.
G. A. HILL

SPECIMEN PAPER WORKED OUT.

1. Simplify

$$
\begin{aligned}
& a-[2b+\{3c-3a-(a+b)\}+\{2a-(b+c)\}] \\
&= a-[2b+\{3c-3a-a-b\}+\{2a-b-c\}] \\
&= a-[2b+3c-3a-a-b+2a-b-c] \\
&= a-2b-3c+3a+a+b-2a+b+c \\
&= 3a-2c.
\end{aligned}
$$

Ans. $3a-2c$.

2. Resolve into factors

$$
\begin{aligned}
& x^2-y^2+z^2-a^2-2xz+2ay \\
&= (x^2-2xz+z^2)-(a^2-2ay+y^2) \\
&= (x-z)^2-(a-y)^2 \\
&= \{(x-z)-(a-y)\}\{(x-z)+(a-y)\} \\
&= (x-z-a+y)(x-z+a-y).
\end{aligned}
$$

3. Find the H.C.F. of

$$
\begin{aligned}
& 5x^2(12x^3+4x^2+17x-3) \\
& \quad \text{and } 10x(24x^3-52x^2+14x-1).
\end{aligned}
$$

$$
\begin{array}{r|r|l}
5x^2(12x^3+4x^2+17x-3) & 10x(24x^3-52x^2+14x-1) & \text{Reserve } 5x \\
12x^3+4x^2+17x-3 & 24x^3-52x^2+14x-1 & 2 \\
\underline{12x^3+4x^2-\quad x} & \underline{24x^3+\ 8x^2+34x-6} & \\
3\overline{)18x-3} & -5\overline{)-60x^2-20x+5} & \\
6x-1 & 12x^2+\ 4x-1 & x \\
 & \underline{12x^2-\ 2x\qquad} & 2x+1 \\
 & 6x-1 & \\
 & \underline{6x-1} &
\end{array}
$$

Ans. $5x(6x-1)$.

4. Simplify

$$\frac{3}{1-2x}-\frac{7}{1+2x}-\frac{4-20x}{4x^2-1}$$
$$=\frac{3}{1-2x}-\frac{7}{1+2x}-\frac{4-20x}{4x^2-1}$$
$$=\frac{3}{1-2x}-\frac{7}{1+2x}+\frac{4-20x}{1-4x^2}.$$

L.C.D. $=1-4x^2$.

$3+6x=$ first numerator,
$-7+14x=$ second numerator,
$4-20x=$ third numerator.

$0=$ sum of numerators. *Ans.* 0.

5. Solve

$$10x+15y-24z=41 \quad (1)$$
$$15x-12y+16z=10 \quad (2)$$
$$18x-14y-7z=-13. \quad (3)$$

Multiply (1) by 2, $20x+30y-48z=82$
Multiply (2) by 3, $45x-36y+48z=30$
Add, $65x-6y=112$ (4)
Multiply (2) by 7, $105x-84y+112z=70$
Multiply (3) by 16, $288x-224y-112z=-208$
Add, $393x-308y=-138$ (5)
Multiply (4) by 154, $10010x-924y=17248$
Multiply (5) by 3, $1179x-924y=-414$
Subtract, $8831x=17662$
$\therefore x=2.$
Substitute value of x in (4), $130-6y=112.$
$\therefore y=3.$
Substitute values of x and y in (1), $20+45-24z=41.$
$\therefore z=1.$

Ans. $x=2,\ y=3,\ z=1.$

6. A passenger train, after travelling an hour, is detained 15 minutes; after which it proceeds at three-fourths of its former rate, and arrives 24 minutes late. If the detention had taken place 5 miles farther on, the train would have been only 21 minutes late. Determine the usual rate of the train.

Let $\qquad x =$ usual rate of train per hour,

and $\qquad y =$ number of miles train has to run.

Then $y - x =$ number of miles train has to run after detention,

$\frac{y-x}{x} =$ number of hours usually required to run $y - x$ miles.

and $\frac{y-x}{\frac{3}{4}x} =$ number of hours actually required to run $y - x$ miles,

Since the detention was 15 minutes, and the train was 24 minutes late, the loss in running-time is 9 minutes, or $\frac{3}{20}$ of an hour.

$$\therefore \frac{y-x}{\frac{3}{4}x} - \frac{y-x}{x} = \frac{3}{20} \qquad (1)$$

If the detention had occurred 5 miles farther on, the loss in running-time would have been 6 minutes, or $\frac{1}{10}$ of an hour.

$$\therefore \frac{y-x-5}{\frac{3}{4}x} - \frac{y-x-5}{x} = \frac{1}{10} \qquad (2)$$

Simplify (1), $\qquad 20y - 29x = 0$

Simplify (2), $\qquad 20y - 26x = 100$

Subtract (1) from (2), $\qquad 3x = 100$

$$\therefore x = 33\tfrac{1}{3}.$$

Ans. $33\frac{1}{3}$ miles.

7. Solve

$$\frac{1}{2x^2+x+1} + \frac{1}{2x^2-3x+1} = \frac{a}{2bx-b} - \frac{2bx+b}{ax^2-a},$$

$$\frac{1}{(2x-1)(x+1)} + \frac{1}{(2x-1)(x-1)} = \frac{a}{b(2x-1)} - \frac{2bx+b}{a(x-1)(x+1)}.$$

$$\text{L.C.D.} = ab(x-1)(x+1)(2x-1).$$

Simplify, $\quad abx - ab + abx + ab = a^2x^2 - a^2 - 4b^2x^2 + b^2,$

$$2abx = a^2x^2 - a^2 - 4b^2x^2 + b^2,$$

$$4b^2x^2 - a^2x^2 + 2abx = b^2 - a^2,$$

$$(4b^2 - a^2)x^2 + 2abx = b^2 - a^2.$$

Complete the square, multiplying by 4 times the coefficient of x^2 and adding the square of the coefficient of x,

$$4(4b^2-a^2)^2x^2 + (\ ) + (2ab)^2 = 16b^4 - 16b^2a^2 + 4a^4,$$

$$(4b^2-a^2)2x + 2ab = \pm(4b^2 - 2a^2),$$

$$(4b^2-a^2)2x = 4b^2 - 2ab - 2a^2,$$

$$\text{or } 2a^2 - 2ab - 4b^2.$$

$$x = \frac{2b^2 - ab - a^2}{4b^2 - a^2}, \text{ or } \frac{a^2 - ab - 2b^2}{4b^2 - a^2}.$$

Ans. $x = \frac{b-a}{2b-a}$, or $-\frac{b+a}{2b+a}$.

8. Solve

$$x + y = 4 \qquad (1)$$
$$x^4 + y^4 = 82 \qquad (2)$$

Put $u + v$ for x, and $u - v$ for y.

(1) becomes $2u = 4$.

$\therefore u = 2.$

(2) becomes $u^4 + 16u^2v^2 + v^4 = 41 \qquad (3)$

Substitute 2 for u in (3), $16 + 24v^2 + v^4 = 41,$

$$v^4 + 24v^2 = 25,$$
$$v^4 + (\ ) + 144 = 169,$$
$$v^2 + 12 = \pm 13,$$
$$v^2 = 1 \text{ or } -25,$$
$$v = \pm 1 \text{ or } \pm\sqrt{-25}.$$

Ans. $\begin{cases} x = 3,\ 1, \text{ or } 2 \pm \sqrt{-25}, \\ y = 1,\ 3, \text{ or } 2 \mp \sqrt{-25}. \end{cases}$

9. Show that $2\sqrt[3]{a^3b^2}$, $\sqrt[3]{ab^5}$ $\frac{1}{2}\sqrt[3]{\frac{a^6}{b}}$ are similar surds.

$$2\sqrt[3]{a^3b^2} = 2a\sqrt[3]{b^2},$$
$$\sqrt[3]{ab^5} = 2b\sqrt[3]{b^2},$$
$$\tfrac{1}{2}\sqrt[3]{\frac{a^6}{b}} = \tfrac{1}{2}\sqrt[3]{\frac{a^6b^2}{b^3}} = \frac{a^2}{2b}\sqrt[3]{b^2}.$$

Since they all have the same surd factor they are similar surds.

10. Simplify

$$(2ab)^{\frac{1}{2}} \times (3ab^2)^{\frac{1}{3}} \div (5ab^3)^{\frac{1}{6}}$$
$$= (2ab)^{\frac{3}{6}} \times (3ab^2)^{\frac{2}{6}} \div (5ab^3)^{\frac{1}{6}}$$
$$= \sqrt[6]{(2ab)^3} \times \sqrt[6]{(3ab^2)^2} \div \sqrt[6]{5ab^3}$$
$$= \sqrt[6]{2^3a^3b^3} \times \sqrt[6]{3^2a^2b^4} \div \sqrt[6]{5ab^3}$$
$$= \sqrt[6]{2^3 \times 3^2 \times a^5 \times b^7} \div \sqrt[6]{5ab^3}$$
$$= \sqrt[6]{\frac{2^3 \times 3^2a^5b^7}{5ab^3}}$$
$$= \sqrt[6]{\frac{2^3 \times 3^2 \times 5^5a^4b^4}{5^6}}$$
$$= \tfrac{1}{5}\sqrt{225000a^4b^4}.$$

Ans. $\frac{1}{5}\sqrt{225000a^4b^4}.$

11. Expand $\left(\dfrac{2x^2}{y} - \sqrt[3]{y^2}\right)^6$.

$$\left(\frac{2x^2}{y} - \sqrt[3]{y^2}\right)^6 = (2x^2y^{-1} - y^{\frac{2}{3}})^6,$$

$$\begin{aligned}(2x^2y^{-1} - y^{\frac{2}{3}})^6 &= (2x^2y^{-1})^6 - 6(2x^2y^{-1})^5(y^{\frac{2}{3}}) + 15(2x^2y^{-1})^4(y^{\frac{2}{3}})^2 \\ &\quad - 20(2x^2y^{-1})^3(y^{\frac{2}{3}})^3 + 15(2x^2y^{-1})^2(y^{\frac{2}{3}})^4 \\ &\quad - 6(2x^2y^{-1})(y^{\frac{2}{3}})^5 + (y^{\frac{2}{3}})^6 \\ &= 64x^{12}y^{-6} - 192x^{10}y^{-\frac{13}{3}} + 240x^8y^{-\frac{8}{3}} - 160x^6y^{-1} \\ &\quad + 60x^4y^{\frac{2}{3}} - 12x^2y^{\frac{7}{3}} + y^4.\end{aligned}$$

Ans. $64x^{12}y^{-6} - 192x^{10}y^{-\frac{13}{3}}$, etc.

12. Find the value of

$$\sqrt[5]{\frac{3.1416 \times 4771.21 \times 2.7183^{\frac{1}{2}}}{30.103^4 \times 0.4343^{\frac{1}{2}} \times 69.897^4}}.$$

$$\begin{array}{lllr} \log & 3.1416 = & 0.4971 & = 0.4971 \\ \log & 4771.21 = & 3.6786 & = 3.6786 \\ \frac{1}{2}\log & 2.7183 = & 0.4343 \div 2 & = 0.2172 \\ 4 \text{ colog} & 30.103 = & (8.5214 - 10) \times 4 & = 4.0856 - 10 \\ \frac{1}{2}\text{ colog} & 0.4343 = & 0.3622 \div 2 & = 0.1811 \\ 4 \text{ colog} & 69.897 = & (8.1555 - 10) \times 4 & = 2.6220 - 10 \\ \hline & & & 11.2816 - 20 \\ & & & 30 \quad - 30 \\ \hline & & 5 \,)\!\! & 41.2816 - 50 \\ \hline & & & 8.2563 - 10 \end{array}$$

Ans. 0.01804.

EXAMINATION MANUAL.

ALGEBRA.

1.

1. If $a=1$, $b=3$, $c=5$, $d=0$,
   find the value of $a^2+2b^2+3c^2+4d^2$.
2. Add $a+b-2c$, $8a+4c+2b$, $3c-2a-6b$,
   and $2b-2c+3a$.
3. Multiply $1+x+x^4+x^5$ by $1-x+x^2-x^3$.
4. Resolve into the simplest factors $a^2x^2y-b^2xy^2$.
5. Find the L.C.M. of xy, $x-y$, and y^3-x^2y.
6. Simplify $\frac{7x-10}{5}-\frac{3x-7}{6}-\frac{27x-30}{30}$.
7. Solve $2(x-3)+6(x-2)=54$.
8. Divide a line 12 inches long into two parts, such that the greater shall be five times the less.

2.

1. If $a=1$, $b=-1$, $c=0$, $d=2$,
   find the value of $(ac-bd)(ad-bc)(ab-cd)$.
2. Divide $a^2+ab+2ac-2b^2+7bc-3c^2$ by $a-b+3c$.
3. Resolve into the simplest factors, $a^2x^2-a^2y^2$.
4. Find the H.C.F. of $9x^2-25$ and $9x^2+3x-20$.
5. Add $\frac{a-b}{ab}$, $\frac{a-c}{ac}$, and $\frac{b-c}{bc}$.
6. Solve $5-3(4-x)+4(3-2x)=0$.
7. A and B together have \$4900. A invests $\frac{1}{6}$ of his money in business, and B $\frac{1}{5}$ of his, and each has the same sum remaining. How much money had each?
8. Solve $\left.\begin{array}{r}2x+3y=22\\3x+4y=31\end{array}\right\}$.

3.

1. If $a=1,\ b=0,\ c=-1$,
evaluate $(1-ab)(1-ac)(1-bc)$.
2. Write down the product of $x+\frac{a}{2},\ x-\frac{a}{2},\ x^2+\frac{a^2}{4}$.
3. Simplify $3a-\frac{1}{2}[b+\{2a-(b-x)\}]$.
4. Find the H.C.F. of $6a^6-6a^3x^3$ and $8a^6-8a^5x$.
5. Subtract $\dfrac{x}{a^2-ax}$ from $\dfrac{a}{ax-x^2}$.
6. Solve $\frac{2}{7}x+\frac{1}{6}(x-1)=x-4$.
7. Nine years ago A was three times as old as B, but now he is only twice as old. Find the present ages of A and B.
8. Solve $\left.\begin{array}{l}\dfrac{x}{3}+\dfrac{y}{2}=\dfrac{7}{6}\\ \dfrac{3x}{7}-\dfrac{2y}{3}=\dfrac{4}{21}\end{array}\right\}$.

4.

1. Multiply $a-b+c$ by $a+b-c$, and find the value of the product when $a=9,\ b=4,\ c=3$.
2. From $5a+3c-4b-7d-e$
take $4a+7d+5e-5b-6c$.
3. Divide x^6-y^6 by $x-y$.
4. Solve $\dfrac{5x}{8}-\dfrac{11x}{12}=\dfrac{7x}{9}-(x+5)$.
5. Solve $\left.\begin{array}{r}7x+4y=17y\\ 6x-10y=8\end{array}\right\}$.
6. Resolve into factors $y^2+25y-150$.
7. Simplify $\dfrac{a^2-4}{a^2+5a}\times\dfrac{a^2-25}{a^2+2a}$.
8. Find a number such that the sum of its fifth and its seventh parts shall exceed the difference of its fourth and its seventh parts by 99.

5.

1. Simplify $3(x+a)(y-b)-\{-a[b-(c-d)]\}$.
2. Find the H.C.F. of $12x^4-108x^2$ and $16x^4-48x^3$.
3. Find the L.C.M. of $4ab(a^2-b^2)^3$ and $6a^3b^2-6a^2b^3$.
4. Multiply $\dfrac{x^2+x-2}{x^2-2x-3}$ by $\dfrac{x^2-x-2}{x^2+2x-3}$.
5. Solve $\dfrac{1}{x}+\dfrac{1}{2x}-\dfrac{1}{3x}=\dfrac{7}{3}$.
6. What sum is that which is as much greater than \$20 as its half is less than \$20?
7. Two trains travelling, one at 26 and the other at 30 miles an hour, start at the same time from two places 120 miles apart, and move towards each other. How long will it be before the trains meet?

6.

1. Add $2a-3b+c$, $2b-3c+a$, $2c-3a+b$, and $a+b+c$.
2. From $5a^3-7a^2b+6ab^2-b^3+5$ take $3a^3+4a^2b-3+8ab^2-3b^3$.
3. Multiply $x^3-2x^2y+2xy^2-y^3$ by $x^2+2xy+y^2$.
4. Divide $a^3-b^3-c^3-3abc$ by $a-b-c$.
5. Solve $\frac{1}{3}(x-4)+\frac{1}{4}(x+4)=\frac{1}{12}(x+20)$.
6. Simplify $\dfrac{x}{1+x}-\dfrac{1}{1-x}-\dfrac{x^2}{1-x^2}$.
7. Solve $\left.\begin{aligned} x+2y+3z&=10\\ 2x+3y+4z&=16\\ 4x+4y+5z&=25 \end{aligned}\right\}$.
8. A man bought 3 horses and 5 cows, and gave the same sum for the 3 horses as for the 5 cows. If he had bought 4 horses and 10 cows, his outlay would have been \$600 more. Find what his outlay was.

7.

1. If $a=1$, $b=3$, $c=5$, $d=0$, find the value of
$$\frac{12a^3-b^2}{3a^2}+\frac{2c^2}{a+b^2}+\frac{a+b^2+c^3}{b^2-2bc+c^2}+\frac{a^2-d^2}{6a-c}.$$
2. Simplify $2-3x-(4-6x)-\{7-(9-2x)\}$.
3. Multiply $1+2x+3x^2$ by $1+4x+5x^2+6x^3$.
4. Find the continued product of
$x+y$, $x-y$, and $x^2+2xy+y^2$.
5. Divide $x^3+(a+b+c)x^2+(ab+bc+ca)x+abc$ by $x+a$.
6. Reduce to the simplest form $\frac{a^2+ab+b^2}{a^2-ab+b^2}\times\frac{a^3+b^3}{a^3-b^3}$.
7. Solve the equation $\frac{x}{4}-\frac{x-5}{3}=\frac{x-2}{6}$.
8. A is twice as old as B and four years older than C. The sum of the ages of A, B, and C is 96 years. Find B's age.

8.

1. Find the value of $\frac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}$ if $a=4$, $b=\frac{1}{2}$, $c=1$.
2. What number diminished by 42 becomes 18?
3. Simplify $\{y^2-(1-y)\}x-\{x^2+(x-y)y\}$.
4. Multiply px^2+qx-r by $mx-n$, and bracket coefficients of the powers of x in the product.
5. Divide $1+x^3-8y^3+6xy$ by $1+x-2y$.
6. Solve $\frac{3x-2}{3}+\frac{90x-9}{9}=5x+4\frac{1}{3}$.
7. Find the length and breadth of a rectangular field if the length is 140 yards more than the breadth, and if it requires 1000 yards of fence to enclose it.
8. Point out first the factors, secondly the terms, and thirdly the coefficients, in the expression $5ax-2ay+abz$.

9.

1. If $a=1$, $b=2$, find the value of
$$a^6-3a^5b+6a^4b^2-7a^3b^3+6a^2b^4-3ab^5+b^6.$$
2. Divide $x^{4a}+4y^{4b}$ by $x^{2a}+2x^ay^b+2y^{2b}$.
3. Resolve into the simplest factors
$$ax^2-a;\ x^4-81a^4;\ 8a^3+27;\ x^2+7x+10.$$
4. Find the L.C.M. of $a+b$, $b-c$, $c+a$, a^2-b^2, b^2+c^2, c^2-a^2.
5. Find the H.C.F. of
$$x^2-y^2+xz+yz \text{ and } x^2+y^2+xz-yz-2xy.$$
6. Simplify $2(x-y)-2\left\{3(x-y)-\left(2x-3y-\frac{x-4}{2}\right)\right\}$.
7. Solve $2a(3a-x)-3x(2b-a)=ab$.
8. The sum of two numbers is 70, and their difference is 23; find the numbers.

10

1. If $a=25$, $b=9$, $c=4$, $d=1$, find the value of
$$3\sqrt{a}+2\sqrt{4b}-\sqrt{9c}+\sqrt{16d}.$$
2. From $7x^3-6x^2+2x-1$ take $4x^3-3x^2-x+2$.
3. Multiply $a^{2n}+a^nx^n+x^{2n}$ by $2a^n-2x^n$.
4. Divide $x^4-9x^2-6xy-y^2$ by x^2+y+3x.
5. Resolve into the simplest factors
$$x^{16}-y^{16};\ 8x^6+\frac{1}{x^3};\ 16x^{12}-y^4;\ x^2+6x-7.$$
6. Find the H.C.F. of
$$6a^2+7ax-3x^2 \text{ and } 6a^2+11ax+3x^2.$$
7. Solve $\dfrac{2x+a}{b}-\dfrac{(x-b)}{a}=\dfrac{3ax+(a-b)^2}{ab}$.
8. A is now twice as old as B, but 10 years ago he was three times as old. Find the present ages of A and B.

11.

1. If $a=0$, $b=2$, $c=4$, $d=6$, find the value of
$$3\sqrt[3]{2b^2-a}+2\sqrt[3]{b^2+c^2+7}-\sqrt[3]{2(b+c)^2-(b+d)^2}.$$
2. From $\frac{5}{2}x^4+\frac{1}{2}x^2+3x-1\frac{3}{4}$ take $\frac{1}{2}x^4-\frac{3}{2}x-2\frac{1}{4}$.
3. Divide $1-6x^5+5x^6$ by $1-2x+x^2$.
4. Simplify $(x+y)^2-2x(3x+2y)-(y-x)(-x+y)$.
5. Resolve into the simplest factors
$$2x^2-3x-5 \text{ and } a^2-b^2+c^2-2ac.$$
6. Find the H.C.F. of
$$y^4+m^2y^2+m^4 \text{ and } y^4+my^3-m^3y-m^4.$$
7. Solve $\frac{x}{2}-\frac{2x-11}{3}-\frac{x+3}{4}=0$.
8. A person bought 16 yards of cloth, and if he had bought one yard less for the same money, each yard would have cost 25 cents more. What did the cloth cost per yard?

12.

1. Find the H.C.F. of
$$12a^2+13ab+3b^2 \text{ and } 6a^2+23ab+7b^2.$$
2. Simplify $2(x-y)-2\left\{3(x-y)-\left(2x-3y-\frac{x-4}{2}\right)\right\}$.
3. Find the square root of
$$9a^4-12a^3b+34a^2b^2-20ab^3+25b^4.$$
4. A is twice as old as B; 22 years ago he was four times as old; what is A's age?
5. Multiply $x^2+\frac{3xy}{4}-y^2$ by $x^2-\frac{3xy}{4}+y^2$.
6. Solve $x+\frac{11-x}{3}=\frac{19-x}{2}$.
7. What fraction is that to the numerator of which if 7 be added, its value is $\frac{2}{3}$; but if 7 be taken from the denominator, its value is $\frac{3}{8}$?
8. Solve $x+2y=23$, $3x+4z=57$, $5y+6z=94$.

13.

1. Add $5x^3 - 3x + 2y$, $-x^3 + 2x - y$, and $7x^3 - 4x + 3y$.
2. From $2x + 11a + 10b - 5c - 23$ take $2c - 10 + 5a - 3b$.
3. Multiply $9x^2 + 3xy + y^2 - 6x + 2y + 4$ by $3x - y - 2$.
4. Divide $x^3 - 5x^2 - x + 14$ by $x^2 - 3x - 7$.
5. Solve $\frac{1}{2}(x+1) + \frac{1}{3}(x+2) - 16 + \frac{1}{4}(x+3) = 0$.
6. Resolve into factors $2xy - x^2 - y^2 + z^2$.
7. Simplify $\frac{x}{y} + \frac{y}{x+y} + \frac{x^2}{x^2+xy}$.
8. Solve $\left.\begin{aligned} 3x - 2y &= 5 \\ 4x - 3y + 2z &= 11 \\ x - 2y - 5z &= -7 \end{aligned}\right\}$.

14.

1. Find the square root of $x^4 + 2x^3 - x + \frac{1}{4}$.
2. Find the H.C.F. of $x^4 + 3x^3 - 6x - 4$ and $x^4 - 3x^3 + 6x - 4$.
3. Resolve into factors $x^2 + 20x + 91$.
4. Divide 86 into two parts such that their difference shall be 18.
5. Simplify $\frac{a-b}{b} + \frac{2a}{a-b} - \frac{a^3 + a^2b}{a^2b - b^3}$.
6. Divide $1 - \frac{x^2}{x^2+a^2}$ by $1 + \frac{a^2}{x^2 - a^2}$.
7. Solve $\left.\begin{aligned} x + y + z &= 90 \\ 2x + 40 &= 3y + 20 \\ 2x + 40 &= 4z + 10 \end{aligned}\right\}$.

15.

1. Multiply $a^3 - 2a^2b + 3ab^2 + 4b^3$ by $a^2 - 2ab - 3b^2$.
2. Divide $x^4 + (2b^2 - a^2)x^2 + b^4$ by $x^2 + ax + b^2$.
3. Find the square root of $\frac{4a^2}{b^2} + 8 + \frac{4b^2}{a^2}$.
4. Solve $(3x-5)(6x-7) - 2(3x-5)^2 = 7$.
5. Solve $\left.\begin{aligned} 3x - y + z &= 17 \\ 5x + 3y - 2z &= 10 \\ 7x + 4y - 5z &= 3 \end{aligned}\right\}$.
6. Resolve into factors $x^2 - y^2 + z^2 - a^2 - 2xz + 2ay$.
7. Simplify
$$\frac{1}{(2-m)(3-m)} - \frac{2}{(m-1)(m-3)} + \frac{1}{(m-1)(m-2)}.$$

16.

1. A body moves a feet toward the north and then b feet towards the south. How many feet is the body, at the end of the motion, from the starting point? On which side of the starting point is the body (i.) if $a > b$, (ii.) if $a < b$?
2. Reduce to the simplest form
$$1 - \{1 - (1 - 4x)\} + \{2x - (3 - 5x)\} - \{2 - (-4 + 5x)\}.$$
3. Divide $a^2 + ab + 2ac - 2b^2 + 7bc - 3c^2$ by $a - b + 3c$.
4. Resolve into elementary factors $a^6 - 9a^4b^6$.
5. Find by inspection the H.C.F. of
$4(x^3 + a^3)$ and $6(x^2 - 2ax - 3a^2)$.
6. Simplify $\frac{x+y}{y} - \frac{2x}{x+y} + \frac{x^2y - x^3}{x^2y - y^3}$.
7. Solve $\frac{1}{12}(2x-3) - \frac{1}{3}(3x-2) = \frac{1}{8}(4x-3) - 3\frac{5}{24}$.
8. A and B have together \$8, A and C have \$10, B and C have \$12. What have they each?

17.

1. Resolve into four factors $(a^2-b^2-c^2)^2-4b^2c^2$.

2. Find the H.C.F. of $4x^3-xy^2$ and $4x^3-10x^2y+4xy^2$.

3. Simplify $3a-(a-b-c)-2\{a+c-2(b-c)\}$.

4. Divide $x^8+x^4y^4+y^8$ by $x^4+x^2y^2+y^4$.

5. Solve $$\left.\begin{aligned} 2x-\frac{y+3}{4}&=7+\frac{3y-2x}{5} \\ 4y+\frac{x-2}{3}&=26\tfrac{1}{2}-\frac{2y+1}{2} \end{aligned}\right\}$$

6. Divide 91 into two such parts that the quotient of the greater part divided by the difference between the parts may be 7.

7. Extract the square root of $16x^4-32x^3+24x^2-8x+1$.

18.

1. From $ax^2+bxy+cy^2$
   take $(b-c)x^2+(c-a)xy+(a-b)y^2$.

2. Multiply $x^2-(a-b)x-ab$ by $x^2+(a-b)x-ab$.

3. Resolve into the simplest factors
   $32x^5+y^5$; $5x^2+8xy-21y^2$; $6a^2x^2-7ax-3$.

4. Simplify $3a-\{2a-(3a-b)^2\}+3a\left\{2b-3a-\dfrac{b^2}{3a}\right\}$.

5. Find the L.C.M. of $3(1-x^2)$, $6(1-x)^2$, $5(x+x^2)$.

6. Find the H.C.F. of $7x^2-12x+5$ and $2x^3+x^2-8x+5$.

7. Solve $3x-\frac{1}{2}(x-1\frac{1}{2})=9-\frac{1}{4}(5x-7)$.

8. If A can do a piece of work in 8 days, and A and B together can do it in 6 days, how long would B take to do it alone?

19.

1. From $ax^{m+n}+(a+b)x^m+2bx^{m-n}$
   take $bx^m+(a+b)x^{m-n}-ax^{m-2n}$.
2. Find the coefficient of x^3 in the product of
   $x^3-bx^2-3abx+a^2$ and $2x^2-3ax-b^2$.
3. Reduce to lowest terms $\dfrac{a^2+b^2-c^2+2ab}{a^2-b^2-c^2+2bc}$.
4. Simplify $\dfrac{a}{a+b}+\dfrac{b}{a-b}-\dfrac{a^2+b^2}{a^2-b^2}$.
5. Find the H.C.F. of
   $7a^2-23ab+6b^2$ and $5a^3-18a^2b+11ab^2-6b^3$.
6. Reduce to a single fraction $a-b+\dfrac{1}{1+\dfrac{b}{4a}}$.
7. Solve $\dfrac{2x}{a-2b}-3=\dfrac{x}{2a-b}$.
8. A square floor would contain 17 square yards more if each side were 1 yard longer. Find its area.

20.

1. Divide $2a^{3n}-6a^{2n}b^n+6a^nb^{2n}-2b^{3n}$ by a^n-b^n.
2. Find the H.C.F. of $20x^5+x^3-x$ and $25x^5+5x^4-x^2-x$.
3. Simplify $\dfrac{2ax+3a^2}{4x^2-3ax}\times\dfrac{4ax^3-3a^2x^2}{a^2x^2-a^4}\times\dfrac{a^2-x^2}{2x^2+3ax}$.
4. Resolve into factors $256x^8-1$ and $x^2-7x-18$.
5. Simplify $\left(\dfrac{x}{x+y}+\dfrac{y}{x-y}\right)\div\left(\dfrac{x}{x-y}-\dfrac{y}{x+y}\right)$.
6. Solve $(a+x)(b+x)-a(b+c)=\dfrac{a^2c}{b}+x^2$.
7. A laborer was engaged for 36 days on condition that for every day he worked he was to receive \$3, and for every day he was idle to forfeit \$2. At the end of the time he received \$68. How many days did he work?

21.

1. Resolve into their simplest factors

   $a^3 - a^2x - ax^2 + x^3$; $x^3 + 3x^2y - 4xy^2 - 12y^3$.

2. Simplify $\dfrac{x+1}{2x-1} - \dfrac{x-1}{2x+1} - \dfrac{1-3x}{x(1-2x)}$.

3. Simplify $\dfrac{1}{a(a+b)} + \dfrac{2b}{a(a^2-b^2)} + \dfrac{1}{b(a-b)}$.

4. Solve $\dfrac{8x+5}{14} + \dfrac{7x-3}{6x+2} = \dfrac{16x+15}{28} + \dfrac{2\frac{1}{4}}{7}$.

5. Find the L.C.M. of

   $8x^3 - 14x + 6$, $4x^2 + 4x - 3$, and $4x^2 + 2x - 6$.

6. Add $1 - \{2 - (3 - x)\}$, $3x - (4 - 5x)$, $4 - (-5 + 6x)$.

7. Twenty-four persons subscribed the cost of a new boat, but four of the subscribers proving defaulters, each of the others had to pay \$2 more than his due share. Find the cost of the boat.

22.

1. Resolve into their simplest factors

   $a^4x^4 - \frac{81}{16}y^4$; $27bx^4 - b^4xy^3$; $20ax^2 + 18a^2x - 18a^3$.

2. Simplify $\dfrac{\left(x + \dfrac{1}{x}\right)^2 - 2\left(1 + \dfrac{1}{x^2}\right)}{\left(x - \dfrac{1}{x}\right)^2}$.

3. Find the L.C.M. of $x^2 - 4y^2$ and $x^2 + xy - 6y^2$.

4. Solve $14(7x+5) - [84x - (9x - 21)] = 210$.

5. Solve $\dfrac{2x+1}{3} - \dfrac{4x+2}{3x-9} = \dfrac{2x-5}{3}$.

6. A and B can do a piece of work in 8 days, working together; A working alone can do it in 12 days. In how many days can B do it, working alone?

7. State the distinction between an algebraic identity and an algebraic equation, and give an example of each.

23.

1. If $a=0$, $b=2$, $c=-3$, $d=4$, find the value of
$4(ad-bc)-\{(a-b)-(c-d)\}^2$.

2. Add $\frac{1}{2}a+\frac{2}{3}b-\frac{3}{4}c+\frac{1}{6}d$ and $\frac{2}{3}a-\frac{5}{6}b+c-\frac{1}{4}d$.

3. Divide $x^6-2a^3x^3+a^6$ by $x^2-2ax+a^2$.

4. Reduce to the lowest terms $\dfrac{x^3+a^3}{x^2+2ax+a^2}$.

5. Solve $\left.\begin{aligned}4x+9y&=12\\6x-3y&=\ \ 7\end{aligned}\right\}$.

6. Extract the square root of $x^4-2x^3+\dfrac{3x^2}{2}-\dfrac{x}{2}+\dfrac{1}{16}$.

7. A person walking at the rate of $4\frac{3}{4}$ miles an hour starts $1\frac{1}{2}$ hours after another person who walks only 4 miles an hour. When and where will he overtake him?

24.

1. If $a=1$, $b=3$, $c=5$, $d=7$, find the value of
$a-2b-\{3c-d-[3a-(5b-c-8d)]-2b\}$.

2. Simplify $(x-3)-(3-x)(1-x)-(x-3)(5-2x)$.

3. From $(a+b)^2-2bx$ take $\{(a+b)(a-x)-(a-b)(b-x)\}$.

4. Find the H.C.F. of
$12xy(x^5y-2x^3y^3+xy^5)$, $20(x^8-2x^5y^3+x^2y^6)$, and $8y(x^7-xy^6)$.

5. Solve $\left.\begin{aligned}&\frac{x+y}{12}-\frac{x-y-2}{5}=1\\&9y-5x=10.\end{aligned}\right\}$.

6. Find a number which is as much greater than 63 as its half is less than 93.

25.

1. Resolve into the simplest factors $x^{12}-y^{12}$.
2. Simplify $a^2-(b^2-c^2)-\{b^2-(c^2-a^2)\}+\{c^2-(b^2-a^2)\}$.
3. Find the H.C.F. of $8a^2x^3+12a^3x^2$ and $12x^5+18ax^4$.
4. Divide $\dfrac{a^2+ab+ac+bc}{a^2-ac+ad-cd}$ by $\dfrac{a+b}{a+d}$.
5. Solve $2ax+b=3cx+4a$.
6. Solve $\left.\begin{array}{l}\dfrac{1}{x}+\dfrac{2}{y}=7\\ \dfrac{2}{y}+\dfrac{3}{z}=21\\ \dfrac{3}{z}+\dfrac{4}{x}=19\end{array}\right\}$.
7. The sum of the two digits of a certain number is six times their difference, and the number itself exceeds six times their sum by 3. Find the number.

26.

1. Solve $\left.\begin{array}{r}\dfrac{x}{2}+\dfrac{y}{3}=4\\ \dfrac{y}{2}+\dfrac{z}{8}=4\\ \dfrac{z}{16}-\dfrac{x}{12}=\dfrac{1}{6}\end{array}\right\}$.
2. A person swimming in a stream which runs $1\frac{1}{2}$ miles an hour finds that it takes him four times as long to swim a mile up the stream as it does to swim the same distance down. Find his rate of swimming in still water.
3. Find the H.C.F. of $3x^4-4x^3+1$ and $4x^4-5x^3-x^2+x+1$.
4. Solve $\dfrac{x-7}{2}+\dfrac{x}{9}=\dfrac{x+7}{3}$.
5. Divide $a^2b+(a-b)^2x-2ax^2-x^3$ by $b+x$.
6. Simplify $\dfrac{x^2+y^2}{x^2-y^2}-\dfrac{y}{x-y}+\dfrac{x}{x+y}$.

27.

1. Subtract
$(a+y)x^3-(2a^2-y^2)x^2-2xy^3$ from $a^3y-(2x^2-a)y^2+ax^3$, and arrange your answer in descending powers of y.

2. Divide $a^2-4ab+4b^2$ by $\dfrac{a^2-2ab}{a+2b}$.

3. Solve $\dfrac{a^2}{x}-\dfrac{b^2}{x}=a-b$.

4. Simplify $\dfrac{x^2+xy}{x-y}\div\dfrac{x^4-y^4}{(x-y)^2}$.

5. Find two consecutive numbers, the difference of the squares of which is equal to 51.

6. Two trains pass a station at an interval of 4 hours, travelling respectively at the rates of $11\frac{1}{2}$ and $17\frac{1}{2}$ miles an hour. How far will the slower train have run before it is overtaken by the other?

7. Solve $\frac{1}{2}x+\frac{1}{3}y=16$; $\frac{1}{3}x-\frac{1}{2}y=2$.

28.

1. Divide x^3 by $1-x^2$ to three terms. What is the difference between the answer thus obtained and the true answer?

2. Arrange according to powers of x
$3(a^2-2x)(a-x^3)+2x(a-6x^2)$.

3. Find the H.C.F. of
$6a^3-6a^2y+2ay^2-2y^3$ and $12a^2-15ay+3y^2$.

4. How do you find the L.C.M. of two expressions when their factors cannot be determined by inspection?

5. Simplify $\dfrac{x^3+a^3}{2ax-x^2}\times\dfrac{x^3}{x^2-ax+a^2}\div\dfrac{4a^2x^2+2ax^3}{4a^2-x^2}$.

6. A, B, and C can reap a field in 30 hours. A can do half as much again as B, and B two-thirds as much again as C. How many hours would each require to do the work alone?

7. Solve $x-\frac{1}{7}(y-2)=5$; $4y-\frac{1}{3}(x+10)=3$.

29.

1. Divide $\frac{a+x}{a-x}+\frac{a-x}{a+x}$ by $\frac{a+x}{a-x}-\frac{a-x}{a+x}$.

2. Simplify $\dfrac{\frac{1}{1+x}+\frac{x}{1-x^2}}{\frac{1}{1-x^2}-\frac{x}{1+x}}$.

3. Solve $5x=43-7y$; $11x+9y=69$.

4. Find the H.C.F. of $3x^6-10x^4+15x^2-8x$ and $2x^6-4x^4+2x^2$.

5. Find all the expressions whose L.C.M. is x^3-4xy^2

6. A number consists of two digits. The sum of the digits is 7, and if 27 be subtracted from the number the order of the digits will be reversed. Find the number.

7. Find the value of $\frac{3}{1+x}+\frac{3}{1-x}$, when $x=\frac{1}{2}$.

30.

1. A is x years old; B is 20 years younger. Express in algebraic language the age of B; the sum of their ages; the age of A five years ago; the age of B five years hence.

2. Simplify $(2a-b)^2+2b(a+b)-3a^2-(a-b)^2+(a+b)(a-b)$.

3. Multiply $a-\frac{x^2}{a}$ by $\frac{a}{x}+\frac{x}{a}$.

4. Simplify $\frac{a(x^2+c^2)-2acx}{b(x^2+c^2)-2bcx}$.

5. Find the H.C.F. of
$3a^3-7a^2b-33ab-7ab^2+11b^2+3b^3$ and $3a^2-10ab+3b^2$.

6. Solve $5x+2y-z=2z-x-y=y+z-2x=6$.

7. If a father was four times as old as his son seven years ago, and will be twice as old as his son in seven years more, what is the present age of each?

31.

1. What is meant by a *term*, an *index*, a *factor?* Give examples.

2. Solve $\frac{m}{x}+\frac{n}{y}=a$; $\frac{n}{x}+\frac{m}{y}=b$.

3. Solve $\left.\begin{array}{r} 2x-4y+9z=28 \\ 7x+3y-5z=3 \\ 9x+10y-11z=4 \end{array}\right\}$.

4. Find the H.C.F. of
$2x^2+3xy+6x+9y$ and $3x^2-2xy+9x-6y$.

5. Simplify $\frac{x^4-y^4}{x^2-2xy+y^2}\div\frac{x^2y+y^3}{x^3-y^3}$.

6. Find a fraction such that if 5 be added to the numerator the fraction equals 1, but if 3 be taken from the denominator the fraction equals $\frac{1}{2}$.

7. Arrange according to descending powers of x
$x^2-ax-c^2x^2-bx+bx^3-cx^2+a^2x^3-x^2-cx$.

32.

1. What is a common multiple of two or more quantities? What is meant by their least common multiple?

2. Find the H.C.F. of
$4x^4+22x^3-26x^2-198x-90$ and $4x^3-14x^2-92x-42$.

3. Simplify $\frac{m}{m+n}+\left(\frac{n}{m+n}\right)^2+\frac{mn}{(m+n)^2}$.

4. Solve $(a+x)(b+x)-a(b+c)=\frac{a^2c}{b}+x^2$.

5. Solve $\frac{2x}{5}+\frac{y}{6}=5$; $\frac{5x}{2}-4y=1$.

6. Solve $x+y=16$; $2z-2x=8$; $2y+3z=51$.

7. At an election there were two candidates, and 1296 persons voted; the successful candidate had a majority of 120. How many votes were cast for each?

8. What is the rule for transposing terms in equations? What is the reason for the rule?

33.

1. Solve $\frac{a+x}{a-b}+\frac{a-x}{a+b}=x-\frac{a^2x}{a^2-b^2}$.
2. Solve $x+y=a$; $bx-cy=d$.
3. Solve $5x-3y+2z=19$; $4x+5y-3z=31$; $3x+7y-4z=31$.
4. Simplify $\frac{a+b}{a-b}-\frac{a-b}{a+b}-\frac{4ab}{b^2-a^2}$.
5. Reduce to lowest terms $\frac{21x^2+14xy}{18ax+12ay}$.
6. Divide $x^6-2a^3x^3+a^6$ by $x^2-2ax+a^2$.
7. Simplify $2\{a(a-b)-(a+b)^2\}\{a^2-2b^2\}$.
8. The difference between two numbers is one-fourth of their sum; and the half of the greater diminished by 7 is equal to one-fifth of the less increased by $2\frac{1}{2}$. Find the numbers.

34.

1. Define *factor*, *common factor*, *highest common factor*.
2. Find the H.C.F. and the L.C.M. of
$(2a^2bx+2abx^2)^2$ and $(6a^2b^2-6b^2x^2)^3$.
3. Multiply a^6+a^5-a-1 by $1-a+a^2-a^3+a^4$.
4. Simplify $\dfrac{1}{1+x+\dfrac{1}{1+x+\dfrac{1}{1+x}}}$.
5. Solve $\frac{a}{b^2x}+\frac{b}{a^2x}=a^2+b^2$.
6. Solve $x+y=5(x-y)=10$.
7. A, B, C, and D have \$290 between them. A has twice as much as C, and B has three times as much as D. Also, C and D together have \$50 less than A. Find how much each has.
8. Expand $(2x^2yz^3)^5$; $(-7a^3b^4c^5)^3$; $(-3ab^4c^2)^4$.

35.

1. Simplify $\dfrac{a+b}{(b-c)(a-c)}-\dfrac{b+c}{(a-b)(c-a)}+\dfrac{a+c}{(b-a)(b-c)}$.

2. Simplify $\dfrac{a^3+3a^2b+3ab^2+b^3}{a^3-3a^2b+3ab^2-b^3}\times\dfrac{a^2-2ab+b^2}{a^2+2ab+b^2}$.

3. Solve $\frac{1}{2}x+\frac{1}{3}y=12-\frac{1}{6}z$; $\frac{1}{2}y+\frac{1}{3}z=8+\frac{1}{6}z$; $\frac{1}{2}x+\frac{1}{3}z=10$.

4. Find the L.C.M. of $9a^2-81$, $12a^2-36a$, and $16a^2+48a$.

5. Extract the square root of $x^4-8x^3-26x^2+168x+441$.

6. Two persons start at noon from towns 60 miles apart. One walks at the rate of 4 miles an hour, but stops $2\frac{1}{2}$ hours on the way. The other walks at the rate of 3 miles an hour, without stopping. When and where will they meet?

7. How many different equations must be given when the values of two or more unknown quantities are required? In what sense must the equations differ?

36.

1. Find the value of $x-(\sqrt{x+1}+2)-\dfrac{x-\sqrt[3]{x}}{\sqrt{x-4}}$ when $x=8$.

2. Divide $a^2x^4-c^2y^4-bx^2y(2ax-by)$ by $(ax-by)x-cy^2$.

3. Find the H.C.F. of
$3ax^2+2ay^2-3bx^2-2by^2$ and $3ax^2-2ay^2-3bx^2+2by^2$.

4. Find the L.C.M. of a^2+x^2, a^2-x^2, a^3+x^3, and a^3-x^3.

5. Simplify $\dfrac{a^4-x^4}{a+3}\times\dfrac{1}{a^2-2x-a(a-2)}\times\dfrac{a^2+5a+6}{(a^2+x^2)(a+x)}$.

6. A is 5 years more than twice as old as B, and is also 25 years older than B. Find their present ages.

7. What are simultaneous equations? Give an example.

8. Expand $\left(\dfrac{a}{b}+\dfrac{b}{a}\right)^3$.

37.

1. Simplify $\frac{bc}{(a-b)(a-c)}+\frac{ac}{(b-a)(b-c)}-\frac{ab}{(b-c)(c-a)}$.

2. Simplify $\frac{\left(\frac{a}{b}-\frac{b}{a}\right)\left(\frac{a+b}{a}-\frac{a+b}{b}\right)}{\left(\frac{a}{b^2}-\frac{b}{a^2}\right)\left(\frac{1}{a}-\frac{1}{b}\right)}$.

3. Solve $\frac{1}{x}+\frac{2}{y}+\frac{3}{z}=\frac{23}{20}$; $\frac{1}{y}+\frac{2}{z}+\frac{3}{x}=\frac{77}{60}$; $\frac{1}{z}+\frac{2}{x}+\frac{3}{y}=\frac{38}{30}$.

4. If a rectangular court had its length diminished by 7 feet, and its width increased by 5 feet, it would become an exact square, enclosing the same area. Find the dimensions of the court.

5. Extract the square root of $x^4-x^3+\frac{5x^2}{4}-\frac{x}{2}+\frac{1}{4}$.

6. Expand $\left(\frac{x^2}{y^2}-\frac{y}{x}\right)^3$.

38.

1. Find the product of $\frac{2x}{a}-\frac{3y}{b}$ and $\frac{4x^2}{a^2}+\frac{6xy}{ab}+\frac{9y^2}{b^2}$.

2. Find the H.C.F. of $x^3+6x^2+11x+6$ and $x^3+9x^2+27x+27$.

3. Simplify $\frac{3x^2+1}{x^2-1}-\frac{5x-2}{x+1}+\frac{2x+5}{x-1}$.

4. Extract the square root of $x^4+\frac{2x^3}{3}+\frac{10x^2}{9}+\frac{x}{3}+\frac{1}{4}$.

5. Solve $ax-\frac{b^2}{a}x=a+b$.

6. A and B have each an annual income of $4000. A spends every year $400 more than B. At the end of 4 years their joint savings amount to $4000. What does each spend annually?

7. Expand $\left(2a-\frac{3}{x}\right)^3$.

8. Explain how to clear an equation of fractions, and the reason for the method.

39.

1. Simplify $\dfrac{1}{a(a-b)(a-c)}+\dfrac{1}{b(a-b)(c-b)}-\dfrac{1}{c(c-a)(b-c)}$.
2. Solve $\dfrac{x-3}{7}-\dfrac{x-6}{6}=\dfrac{x+12}{12}-\dfrac{x-6}{3}+\dfrac{x}{8}$.
3. Solve $\dfrac{x}{4}+\dfrac{y}{2}=8;\quad \dfrac{x}{2}+\dfrac{y}{6}=6$.
4. Extract the square root of $\dfrac{a^2}{9b^2}-2+\dfrac{9b^2}{a^2}$.
5. Simplify $1-\{1-(1-4x)\}+\{2x-(3-5x)\}-\{2-(-4+5x)\}$.
6. Find the L.C.M. of a^2-1, $(ab-b)c$, and $(ac+c)b$.
7. A and B can mow 225 square rods in a day; B and C can mow 223 square rods in a day; A and C can mow 230 square rods in a day. How many square rods can each mow by himself?

40.

1. Simplify $3b\{a(a-b)(3a-b)-(a-b)^3\}$.
2. Find the H.C.F. of
   $12a^2+13ab+3b^2$ and $6a^2+23ab+7b^2$.
3. Find the L.C.M. of
   $4(1-x)^2,\ 8(1-x),\ 8(1+x),\ 4(1+x^2)$.
4. Find the cube root of
   $1-6x+21x^2-44x^3+63x^4-54x^5+27x^6$.
5. Simplify $\left\{\dfrac{1}{3}+\dfrac{2a}{3(1-a)}\right\}\times\left\{\dfrac{3}{4}-\dfrac{3a}{2(1+a)}\right\}$.
6. Solve $\dfrac{x}{2}-1=\dfrac{y}{3}+1=\dfrac{z}{4};\ x+y+z=17$.
7. At what time between two and three o'clock are the hour and minute hands of a clock in a straight line and pointing in opposite directions?

41.

1. Find the square root of $1 - xy - \frac{15}{4}x^2y^2 + 2x^3y^3 + 4x^4y^4$.

2. Simplify $\dfrac{\frac{2}{3}(2-\frac{1}{2}a)-a}{2-\frac{1}{2}(2+2a)} - \dfrac{\frac{3}{4}(6a+4)+\frac{1}{2}a}{1+2a} + 2$.

3. Solve $x+y+z=6$; $x+y-z=4$; $x-y+z=2$.

4. Multiply $\dfrac{2x}{3} - \dfrac{1}{2}$ by $\dfrac{x^2}{2} - \dfrac{x}{3} + \dfrac{1}{4}$.

5. A boat is rowed 1 mile in $5\frac{1}{2}$ minutes; another is rowed 1 mile in $5\frac{3}{4}$ minutes. If they start at the same time from opposite ends of a 4-mile course, at what distance from each starting-point will they meet?

6. Expand $\left(\dfrac{a}{2} + \dfrac{2b}{3}\right)^3$.

7. Solve $\dfrac{2x+a}{3(x-a)} + \dfrac{3x-a}{2(x+a)} = 2\frac{1}{6}$.

42.

1. Simplify $\dfrac{x}{x-3} - \dfrac{x-3}{x} + \dfrac{x}{x+3} - \dfrac{x-3}{x}$.

2. Find the H.C.F. of $3x^3-3x^2y+xy^2-y^3$ and $4x^2-xy-3y^2$.

3. Find the cube root of $a^3 - a^2b + \dfrac{ab^2}{3} - \dfrac{b^3}{27}$.

4. Solve $(2+x)(8+x)-22 = \frac{3}{2} + x^2$.

5. Solve the simultaneous equations
$3x = 2(y+2) - \frac{1}{3}$; $2y = 3(z-x) + \frac{3}{2}$; $z = x + y - \frac{2}{3}$.

6. Find the value of $2(1+\sqrt{a}) - \dfrac{\sqrt{x}-\sqrt{a}}{3} + \dfrac{x-a}{\sqrt{a}+\sqrt{x}}$, when $x = 16a$.

7. 36 octavo volumes exactly fill a box whose length and breadth are $1\frac{1}{2}$ feet and $11\frac{1}{3}$ inches, respectively. The dimensions of each volume are 9 inches by $5\frac{2}{3}$ inches by $1\frac{1}{2}$ inches. Find the depth of the box.

43.

1. Find the L.C.M. of $9x^3+53x^2-9x-18$ and $x^2+11x+30$.
2. Find the square root of $x^4-4x^3+x^2+6x+\frac{9}{4}$.
3. Solve $3x+4y-5z=32$; $4x-5y+3z=18$; $5x-3y-4z=2$.
4. Divide $5x^4+\frac{7ax^3}{2}-\frac{107}{12}a^2x^2+\frac{5a^3x}{6}+\frac{7a^4}{6}$ by $2x^2-ax-\frac{a^2}{2}$.
5. Simplify $\dfrac{1}{x+\dfrac{1}{1+\dfrac{x+1}{3-x}}}$.
6. Simplify $\dfrac{3xyz}{yz+xz-xy}-\dfrac{\dfrac{x-1}{x}+\dfrac{y-1}{y}+\dfrac{z-1}{z}}{\dfrac{1}{x}+\dfrac{1}{y}-\dfrac{1}{z}}$.
7. Reduce to lowest terms $\dfrac{3x^3-2x^2-x}{6x^2-x-1}$.

44.

1. Simplify $\dfrac{1}{(2x-1)^2}+\dfrac{1}{(2x+1)(1-2x)}$.
2. Divide $\dfrac{4a^2-4ax}{(a+x)^2}$ by $\dfrac{a^2-x^2}{4ax+4x^2}$.
3. Add together $\frac{a^2}{2}-\frac{b^2}{3}+\frac{c^2}{4}$, $\frac{b^2}{2}-\frac{c^2}{3}+\frac{a^2}{4}$, $\frac{c^2}{2}-\frac{a^2}{3}+\frac{b^2}{4}$.
4. Find the L.C.M. of
   $x^3+x^2y+xy^2+y^3$ and $x^3-x^2y+xy^2-y^3$.
5. Extract the square root of $a^2+4b^2+c^2+2ac+4ab+4bc$.
6. Solve $\dfrac{a}{x-b}=\dfrac{b}{x-a}$.
7. Divide the number 90 into four parts so that the first part increased by 2, the second diminished by 2, the third multiplied by 2, and the fourth divided by 2, may be all equal.
8. What powers of a quantity are the same whether the quantity be positive or negative?

45.

1. Simplify $\dfrac{1}{1+3a+2a^2}+\dfrac{1}{1-a-2a^2}$.

2. Multiply $1-x-x^2$ by x^2-x-1.

3. Find the H.C.F. of
$x^3+5x^2+10x+8$ and x^5+2x^4-x-2.

4. Find the L.C.M. of
$1-x^2,\ 1-2x+x^2,\ 1-x+x^2-x^3,\ 1-x-x^2+x^3$.

5. Reduce to lowest terms $\dfrac{2x^2-xy-y^2}{y^2+xy-2x^2}$.

6. Expand $\left(\dfrac{ab}{4}-\dfrac{1}{ab}\right)^3$.

7. Solve $\dfrac{1}{y}+\dfrac{1}{z}-\dfrac{1}{x}=6$; $\dfrac{1}{z}+\dfrac{1}{x}-\dfrac{1}{y}=4$; $\dfrac{1}{x}+\dfrac{1}{y}-\dfrac{1}{z}=2$.

46.

1. Reduce to a simple fraction $\dfrac{x}{a-\dfrac{x}{a-\dfrac{a}{x}}}$.

2. Reduce to its lowest terms $\dfrac{x^3-3x^2+7x-21}{2x^4+19x^2+35}$.

3. Find the square root of $\dfrac{4x^4-12x^3y+x^2y^2+12xy^3+4y^4}{16x^2y^2}$.

4. Solve $3x-a=\dfrac{a+x}{3}-\dfrac{b-x}{a}-cx$.

5. The united ages of a father and son make 75 years. The father was 25 years old when the son was born. What is the age of each?

6. Solve $\left.\begin{aligned}5x-3y-28&=0\\3x-2y-13&=0\end{aligned}\right\}$.

7. Subtract ax from $\dfrac{2ax+a^2x^2+2x}{2+ax-2x^2}$.

47.

1. Multiply a^6+a^5-a-1 by $1-a+a^2-a^3+a^4$.
2. Find the H.C.F. of
   $4x^4+22x^3-26x^2-198x-90$ and $4x^3-14x^2-92x-42$.
3. Find the L.C.M. of
   $x^2-1,\quad x^3-1,\quad x^3+1,\quad (x+1)^2,\quad (x-1)^2$.
4. Solve $\left.\begin{aligned} x+3y+2z&=11\\ 2x+\ y+3z&=14\\ 3x+2y+\ z&=11\end{aligned}\right\}$.
5. What sum is that which is as much greater than \$2 as its half is less than \$3?
6. A number consisting of two digits when divided by the sum of the digits gives 7 for the quotient; and the number formed by interchanging the digits, when divided by 6, gives 4 for the quotient. Find the number.
7. What powers of a quantity are always positive? What powers of a negative quantity are always negative?

48.

1. Simplify $\dfrac{x+1}{\dfrac{1}{x+1}+\dfrac{1}{x-1}+\dfrac{2}{1-x^2}}$.
2. Simplify
   $\left(\dfrac{x^2-y^2}{x^2-2xy+y^2}\div\dfrac{x^2+3x}{x-3}\right)\times\left(\dfrac{x^5-9x^3}{x+y}\div\dfrac{x^3-3x^2}{x-y}\right)$.
3. Solve $\dfrac{1}{2x^2+x-1}+\dfrac{1}{3x^2+4x+1}$.
4. Reduce to lowest terms $\dfrac{2x^2-3x-2}{5x^2-7x-6}$.
5. Simplify $\dfrac{x^2+5x+4}{x^2+7x+12}\div\dfrac{x^2+2x+1}{x^2+3x+2}$.
6. Simplify $\dfrac{am^2+an^2+m^2n^2+m^4}{bm^2+bn^2+m^2n^2+n^4}$. What would the result be if $a=-m^2$?

49.

1. Solve $\frac{a+b}{2b} - \frac{1}{2}c\left(\frac{a-b}{bx}\right) = \frac{bc}{(a+b)x} + \frac{a}{a+b}$.

2. Solve $ax + by = c$; $a'x + b'y = c'$.

3. Solve $x+y+z=5$; $3x-5y+7z=75$; $9x-11z+10=0$.

4. Solve $\frac{1}{x}+\frac{1}{y}=m$; $\frac{1}{y}+\frac{1}{z}=n$; $\frac{1}{x}+\frac{1}{z}=p$.

5. Solve
$$\frac{x}{3}+\frac{y}{5}+\frac{2z}{7}=58;\ \frac{5x}{4}+\frac{y}{6}+\frac{z}{3}=76;\ \frac{x}{2}-\frac{y}{5}+\frac{7z}{40}=29\tfrac{2}{5}.$$

50.

1. A person walking at the rate of $4\frac{3}{4}$ miles per hour starts $1\frac{1}{2}$ hours after another person who walks only 4 miles an hour. When and where will he overtake him?

2. A number consists of three digits, the right-hand digit being 0. If the left-hand and the middle digits be interchanged, the number will be diminished by 180. If the left-hand digit be halved, and the middle and right-hand digits be interchanged, the number will be diminished by 454. Find the number.

3. A had twice as much money as B; but when B had received $5 from A, he had three times as much as A. How much had they each at first?

4. A tank of water may be filled by three pipes. By the first pipe it can be filled in 4 hours, by the second in 10 hours, by the third in 15 hours. In what time will it be filled if all three pipes are opened?

   What will be the solution of this question if the general symbols m, n, p are used in place of the numbers 4, 10, 15, respectively?

5. What are simultaneous equations? Give an example.

51.

1. Bracket like powers of x in the expression
$$ax^3 - bx^2 - cx - bx^3 + cx^2 - dx + cx^3 - dx^2 - cx.$$

2. Divide $a^3 - b^3 + c^3 + 3abc$ by $a - b + c$.

3. Simplify $\frac{a}{b} - \frac{(a^2 - b^2)x}{b^2} + \frac{a(a^2 - b^2)x^2}{b^2(b + ax)}$.

4. Divide 150 into two parts such that if one is divided by 23, and the other by 27, the sum of the quotients is 6.

5. Solve $\frac{3x^2}{4} - \frac{15x^2 + 8}{6} = 2x^2 - 3$.

6. Solve $2x = 4 + \frac{6}{x}$.

52.

1. Subtract $(a-b)x-(b-c)y$ from $(a+b)x+(b+c)y$.

2. Find the difference in value between the arithmetical expression 57 and the algebraical expression ab, when $a = 5$ and $b = 7$.

3. Find by inspection the H.C.F. of
$$9(a^2x^2 - 4) \text{ and } 12(a^2x^2 + 4ax + 4).$$

4. Simplify $(4a^{-\frac{2}{3}})^{-\frac{3}{2}}$.

5. Solve $\frac{x+2}{x-1} - \frac{4-x}{2x} = \frac{7}{3}$.

6. There is a rectangular field whose length exceeds its breadth by 16 yards. The field contains 960 square yards. Find its dimensions.

53.

1. Find the L.C.M. of
   $4(a^3 - ab^2)$, $12(ab^2 + b^3)$, and $8(a^3 - a^2 b)$.

2. Simplify $\dfrac{1}{2(a-x)} + \dfrac{1}{2(a+x)} + \dfrac{a}{a^2+x^2}$.

3. Simplify $(64c^{16})^{-\frac{5}{6}}$.

4. Divide $16x - y^2$ by $2x^{\frac{1}{4}} - y^{\frac{1}{2}}$.

5. Solve $1 - \dfrac{x+5}{2x+1} = \dfrac{x-6}{x-2}$.

6. Find two numbers in the ratio of $2\frac{1}{2}$ to 2, such that when diminished each by 5 they shall be in the ratio of $1\frac{1}{3}$ to 1.

54.

1. Solve $\left.\begin{aligned} x + 2y &= 7 \\ y + 2z &= 2 \\ 3x + 2y &= z - 1 \end{aligned}\right\}$.

2. The sum of \$330 is laid out in two investments, on one of which 15 per cent is gained, and on the other 8 per cent is lost. The total amount returned from the investments is \$345. Find each investment.

3. Simplify $\{(a^{-3}b^2)^{\frac{1}{2}}\}^{-\frac{2}{3}}$.

4. Divide $x^{-1} - y^{-1}$ by $x^{-\frac{1}{3}} - y^{-\frac{1}{3}}$.

5. Reduce to an *entire* surd the expression $\frac{1}{2}(\frac{3}{4})^{-\frac{2}{3}}$.

6. Solve $\dfrac{2}{x-1} - \dfrac{1}{x+3} = \dfrac{3}{8}$.

55.

1. Find the L.C.M. of
$$6(x^2y+xy^2),\ 9(x^3-xy^2),\ \text{and}\ 4(y^3+xy^2).$$

2. Simplify $\dfrac{x}{1-x}-\dfrac{x^2}{(1-x)^2}+\dfrac{x^3}{(1-x)^3}$.

3. Arrange in order of magnitude $2\sqrt[3]{22},\ 3\sqrt[3]{7},\ 4\sqrt{2}$.

4. Find the cube root of
$$x^6-3ax^5+5a^3x^3-3a^5x-a^6.$$

5. Simplify $3\sqrt[3]{432}$.

6. Solve $\dfrac{5x}{x+4}-\dfrac{3x-2}{2x-3}=2$.

56.

1. Show that $\sqrt{20},\ \sqrt{45},\ \sqrt{\frac{4}{5}}$ are similar surds.

2. Simplify $2\sqrt{ax}\times\sqrt[3]{3a^2b}\times\sqrt{2bx}$.

3. Simplify $\sqrt[3]{128}+\sqrt[3]{686}+\sqrt[3]{16}$.

4. Simplify $\dfrac{2x^2-8x+6}{x^2-5x+4}\times\dfrac{x^2-9x+20}{x^2-10x+21}\times\dfrac{x^2-7x}{2x^2-10x}$.

5. Solve $\frac{1}{3}x^2+\frac{11}{15}x=3\frac{11}{12}$.

6. If each side of a square plot were enlarged by 10 yards, it would contain $1\frac{1}{2}$ acres more. Find the dimensions of the plot. (1 acre $=4840$ sq. yds.)

57.

1. Simplify $\{x^{-\frac{3}{2}}y(xy^{-2})^{-\frac{1}{2}}(x^{-1}y)^{-\frac{2}{3}}\}^3$.
2. Simplify $\sqrt{128}-2\sqrt{50}+\sqrt{72}-\sqrt{18}$.
3. By selling a cow for \$24 I lose as much per cent as the cow cost me. What was the cost of the cow?
4. Solve $\frac{x}{100}+\frac{21}{25x}=-\frac{1}{4}$.
5. Solve $\left.\begin{array}{l} x+y+z=5 \\ x+y=z-7 \\ x-3=y+z \end{array}\right\}$.
6. A person has travelled 3036 miles. He has gone 7 miles by water to 4 on foot, and 5 by water to 2 on horseback. How many miles has he travelled each way?

58.

1. Solve $\frac{27-x}{7}+\frac{3x-4}{5}-\frac{5x-2}{9}=2$.
2. Find by inspection the L.C.M. of
   a^2-1, a^2+2a-3, and a^3-7a^2+6a.
3. A steamer takes two hours and forty minutes less time to travel from A to B than from B to A. The steamer travels at the rate of 14 miles per hour in still water, and the stream flows at the rate of $1\frac{3}{4}$ miles per hour. Find the distance from A to B.
4. Which is the greater $\frac{1}{2}\sqrt{2}$ or $\frac{1}{3}\sqrt[4]{27}$?
5. Solve $\frac{15}{x}-\frac{72-6x}{2x^2}=2$.
6. A and B enter into partnership with a joint capital of \$3400. A put in his money for 12 months; B put in his money for 16 months. In closing the business, A's share was \$2070 including his profit, and B's share \$1920. Find the sum put in by each.

59.

1. Solve $x+ay=b$; $ax-by=c$.
2. If $a=4$, $b=3$, $c=1$, $d=7$, find the value of
$$\frac{a^2+ac+b^2}{a^2-ac+b^2}-\frac{\sqrt{4ab+b^2+d}}{\sqrt{4ab-b^2-2d}}-\frac{c}{a+b+c+d}.$$
3. Find the H.C.F. of
$$x^4+3x^3-6x-4 \text{ and } x^4-3x^3+6x-4.$$
4. Simplify $\dfrac{1}{x-y}+\dfrac{1}{x+y}-\dfrac{2}{x}$.
5. Find the square root of $a^2-4ax+6a+4x^2-12x+9$.
6. When are the hour and minute hands of a clock together between 10 and 11 o'clock?
7. Solve $\frac{1}{3}(x-1)=\frac{1}{2}(x+1)-\frac{1}{6}(x-1)^2$.

60.

1. If $x=8$, $y=1$, find the value of
$$\sqrt[3]{x}\left\{y^3+\sqrt[4]{\frac{x^2+2xy+y^2}{y^2}}\right\}.$$
2. Simplify $\dfrac{x^4-y^4}{x^2-2xy+y^2}\div\dfrac{x^2y+y^3}{x^3-y^3}$.
3. Find the H.C.F. of
$$x^3-3x^2+7x-21 \text{ and } 2x^4+19x^2+35.$$
4. Find the square root of $\left(x^2+\dfrac{x^2}{y}-2x\right)\left(1+\dfrac{1}{y}\right)+1$.
5. Solve $\frac{1}{12}x-\frac{1}{8}(8-x)-\frac{1}{4}(5+x)+\frac{11}{4}=0$.
6. Find a fraction such that if 5 be added to the numerator, it will be equal to 1, but if 3 be taken from the denominator, it will be equal to $\frac{1}{2}$.
7. A boat's crew can row 5 miles an hour in still water. If they row $3\frac{1}{2}$ miles down a river and then back again to the starting-point in 1 hour 40 minutes, what is the velocity of the current?

61.

1. Solve $7x^2 - 11x = 6$.

2. Solve $\frac{x}{a+b} + \frac{x}{a-b} = \frac{4}{a^2-b^2}$.

3. Solve $\frac{x}{a} + \frac{y}{b} = 2$; $bx - ay = 0$.

4. Find the H.C.F. of
   $48x^2 + 16x - 15$ and $24x^3 - 22x^2 + 17x - 5$.

5. Divide $a^3 + b^3 + c^3 - 3abc$ by $a + b + c$.

6. Extract the square root of $x^4 - 2x^3 + \frac{3x^2}{2} - \frac{x}{2} + \frac{1}{16}$.

7. A vessel containing 120 gallons was emptied in 10 minutes by two pipes running *successively;* the first pipe discharged 14 gallons in a minute, and the second pipe discharged 9 gallons in a minute. How many minutes did each pipe run?

62.

1. Subtract $a^2 - 13ab + 10ac - c^2$ from $7a^2 + 12ab - 3ac - b^2$, and find the value of the remainder, when $a = 1$, $b = 2$, $c = -3$.

2. Divide $x^5 - \frac{1}{x^5}$ by $x - \frac{1}{x}$.

3. Simplify $\frac{m+n}{m-n} \div \left[\frac{(m+n)^3 - (m-n)^3}{(m^2-n^2)(3m^2+n^2)}\right]$.

4. Find the H.C.F. of
   $96x^4 + 8x^3 - 2x$ and $32x^3 - 24x^2 - 8x + 3$.

5. Extract the square root of
   $a^2 - 4ab^2 - 6ac^3 + 4b^2(b^2 + 3c^2) + 9c^6$.

6. Solve $5x + \frac{9}{2}y = 52$; $\frac{3}{5}x + 4y = 27$.

7. Solve $5x + \frac{4x+3}{3x-2} = \frac{10x^2-7}{2x-1} + 9$.

8. When are the hour and minute hands of a watch at right angles between 3 and 4 o'clock?

63.

1. Solve $\dfrac{5x+3}{x-1}+\dfrac{2x-3}{2x-2}=9.$

2. Solve $\left.\begin{array}{r} x+2y+3z=10 \\ 2x+3y+4z=16 \\ 4x+4y+5z=25 \end{array}\right\}.$

3. Solve $\dfrac{9x^2+1}{3x-2}-3x=\dfrac{5x-1}{4x-3}+3.$

4. Find the L.C.M. of
$x^3-4x^2+9x-10$ and $x^3+2x^2-3x+20.$

5. Simplify $\dfrac{a+\dfrac{b-a}{1+ba}}{1-\dfrac{a(b-a)}{1+ba}}.$

6. A and B have the same income. A saves $\frac{1}{5}$ of his income, and B spends \$50 a year more than A. At end of the 4 years B finds himself \$100 in debt. Find their income.

64.

1. Add together
$\dfrac{1}{1+\dfrac{1}{1+x}}$, $\dfrac{x-(1-x)}{x+3-(1-3x)}$, and $\dfrac{10x^2-x-3}{4x^2+4x+1}.$

2. Divide $x^4-\frac{13}{6}x^3+x^2+\frac{4}{3}x-2$ by $\frac{4}{3}x-2.$

3. Find the L.C.M. of $a^3-a^2b-ab+b^2$ and $a^4-b^2.$

4. Extract the square root of a^2+b to three terms.

5. Solve $ax^2+bx+c=a(1-x)^2+b(1-x)-c.$

6. Solve $\dfrac{15}{x+\frac{1}{4}}=\dfrac{15}{x}-\dfrac{1}{4}.$

7. Two numbers consist of the same two digits, but in an inverse order. The sum of the two numbers is 55, and their difference is 27. Find the numbers.

65.

1. Simplify $\dfrac{1}{x-1+\dfrac{1}{1+\dfrac{x}{4-x}}}$.

2. Find the L.C.M. of x^2-x, x^2-1, x^3-2x^2, and x^3-4x.

3. Solve $\dfrac{10x-7}{2x-1}-\dfrac{9x-1}{3x+1}=2$.

4. Solve $2x+3y+4z=34$; $3x-2y+2z=13$; $y-z=1$.

5. Simplify $10x-\{3x-[-4x-(-2x+3x)]\}$.

6. Solve $(x+a)(a^2-ax+b^2)=a^3+bx^2$.

66.

1. The sum of two numbers is 1000, and their difference is $\frac{1}{12}$ of the less; find them.

2. Simplify $\dfrac{4x^4-1}{9x^6-y^2}\times\dfrac{3x^3-y}{2x^2-1}$.

3. Find the H.C.F. of $20x^4-x^2-1$ and $25x^4-5x^3-x-1$.

4. Extract the square root of $m^2+2m-1-\dfrac{2}{m}+\dfrac{1}{m^2}$.

5. Solve $\dfrac{x}{y}=\dfrac{a-x}{y-2b}=\dfrac{a}{b}$.

6. Solve $x+\dfrac{1}{2}=\dfrac{1}{2x}$.

7. When a term is transposed from one side of an equation to the other side, the sign of the term must be changed. Why?

67.

1. If $y = \frac{1 - z^2}{1 + z^2}$ and $z = \frac{1 - x}{1 + x}$, find the value of y in terms of x, in its simplest form.

2. Add together
$$1 - \{\tfrac{1}{2} - (\tfrac{1}{4} - x)\},\ \ \tfrac{1}{2} - (\tfrac{1}{3} - 5x),\ \ \tfrac{1}{3} - (\tfrac{1}{4} + 4x).$$

3. Simplify $(a^{p-q})^{p+q} \times (a^q)^{q+r} \div (a^p)^{p-q}$.

4. Solve $x + a + b + c = \frac{x^2 + a^2 + b^2 + c^2}{a + b - c + x}$.

5. Solve $\left.\begin{aligned} \frac{x+y}{3} - 2y &= 2 \\ \frac{2x - 4y}{5} + y &= \frac{23}{5} \end{aligned}\right\}.$

6. A person out walking has 18 miles to go, and finds that at the rate at which he is going he will be half an hour late, but if he quickens his pace by half a mile an hour, he will arrive just at the proper time. At what rate is he going?

68.

1. Solve $ax^2 + b(2a + b)x - (a^3 - b^3) = 0$.

2. Multiply $x^2 - \frac{1}{2}x + \frac{2}{3}$ by $\frac{1}{3}x + 2$.

3. Divide $x^4 + 10x^3 + 35x^2 + 24 + 50x$ by $(x + 1)(x + 4)$.

4. Simplify $\frac{x^2 + xy}{x - y} \div \frac{x^4 - y^4}{(x - y)^2}$.

5. Solve $\left.\begin{aligned} \frac{x}{3} + \frac{y}{5} - 5 &= 0 \\ 2x + \frac{y}{3} - 17 &= 0 \end{aligned}\right\}.$

6. Expand $\left(\frac{a}{3} - \frac{3b}{ac}\right)^3$.

69.

1. What values of x will make the value of the expression $(x-a)(x-b)(x-c)(x-d)$ equal to 0?

2. Resolve into the simplest factors
$$m^2+n^2+p^2+q^2+2(mq-pn).$$

3. Solve $2x-\dfrac{x+3}{3}+15=\dfrac{12x+26}{5}$.

4. Divide $20,000 between A, B, and C so that A's share shall be ½ of C's, and A's and B's together shall be equal to C's.

5. Find the H.C.F. of
$$6a^3-6a^2y+2ay^2-2y^3 \text{ and } 12a^2-15ay+3y^2.$$

6. Solve $ax^2-a^2(x+b^2)=ab(x-ab)$.

70.

1. If $a=3$, $b=1$, $c=-2$, $d=0$, find the value of
$$\frac{a^3-b^3}{2ab+2bc+2cd+2ad}\div\frac{a^2+b^2+c^2+d^2}{\sqrt{(a-c)(b+c)+6(-c-b)(a-2b)}}.$$

2. Divide $(a^2+2bc)^3-4bc(3a^4+4b^2c^2)$ by a^2-2bc.

3. Find the H.C.F. of
$$6x^3-x^2+16 \text{ and } 6x^4-3x^3+9x^2+3x+12.$$

4. Simplify $\dfrac{x^2+x+1}{x-1+\dfrac{1}{x}}\times\dfrac{\dfrac{1}{x^2}-\dfrac{1}{x}+1}{x+1+\dfrac{1}{x}}$.

5. Simplify $\dfrac{1}{x-1}-\dfrac{1}{x+1}-\dfrac{2}{x^2+x+1}$.

6. Solve $\dfrac{1}{y}+\dfrac{1}{z}=7;\ \dfrac{1}{x}+\dfrac{1}{z}=6;\ \dfrac{1}{x}+\dfrac{1}{y}=5$.

7. Solve $\dfrac{15}{x+\frac{1}{4}}=\dfrac{15}{x}-\dfrac{1}{4}$.

71.

1. Subtract $3x-\frac{x^2}{8}-(2x+3x^2)$ from $\frac{x^2}{2}-\frac{5x}{6}-\left(\frac{3x}{4}-x^2\right)$.

2. Simplify $\frac{1}{4(1+x)}+\frac{1}{4(1-x)}+\frac{1}{2(1+x^2)}$.

3. Find the square root of
$$(a+b)^2-c^2+(a+c)^2-b^2+(b+c)^2-a^2.$$

4. There are 1200 books in my library. There are twice as many English books as French books, and five times as many French books as books in other foreign languages. How many English books and how many French books do I have?

5. Solve $\frac{(4a^2-b^2)(x^2+1)}{4a^2+b^2}=2x$.

6. A man walking $\frac{1}{3}$ of a mile above his ordinary rate gains $\frac{3}{4}$ of an hour in 39 miles. At what rate does he ordinarily walk?

72.

1. Divide $\frac{a^2x+ax^2+x^3}{a^3-x^3}$ by $\frac{x}{a-x}$.

2. Extract the square root of $x^2+\frac{y}{2}$ to three terms.

3. Divide $\frac{(a^{\frac{1}{2}}b^{\frac{1}{4}})^3}{\sqrt[3]{c^2}}$ by $\frac{\sqrt[6]{c^2b^5}}{\sqrt[5]{a^7}}$.

4. Solve $x+y+z=0$; $ax+by+cz=0$; $2x+3y+z=1$.

5. Solve $\frac{x}{100}+\frac{21}{25x}=-\frac{1}{4}$.

6. Find the H.C.F. of
$$x^4+5x^3-7x^2-9x-10 \text{ and } 2x^4-4x^3+8x-4.$$

73.

1. Simplify $\sqrt[3]{a^2bc} \times \sqrt[4]{abc^2} \div a^{\frac{11}{12}} b^{\frac{7}{12}} c^{\frac{2}{3}}$.

2. Find the L.C.M. of $x^2 - 9x + 20$ and $x^2 + 6x - 55$.

3. Solve $\dfrac{2x^2+1}{9x^2-16} = \dfrac{x}{4+3x} - \dfrac{1}{9}$.

4. The sum of two numbers exceeds three times their difference by 8, and their difference is equal to $\frac{1}{3}$ of the smaller number. Find the numbers.

5. Solve $\dfrac{x+m-2n}{x+m+2n} = \dfrac{n+2m-2x}{n-2m+2x}$.

6. Extract the square root of
$$8x^2(2x^2+6ax+a) + c^2(36x^2+12x+1).$$

7. Solve $(x+10\frac{1}{3})^2 = 4(x+9\frac{1}{3})$.

74.

1. Two men run a mile. The winner finishes in 4 minutes 44 seconds, and wins by 2 seconds. How many yards start must the loser of the race have for a tie, if each runs at a uniform rate?

2. Find two consecutive numbers, the difference of whose squares is 51.

3. Solve $\frac{1}{5}(12-x) + \dfrac{3x-\frac{1}{2}}{3} = \dfrac{4x-\frac{2}{3}}{2} + \dfrac{3\frac{1}{6}-x}{7}$.

4. Find the H.C.F. of $4x^3+4x^2-9x-9$ and x^3+2x^2-x-2.

5. Simplify $\dfrac{\dfrac{1}{a+b}+\dfrac{1}{a-b}}{\dfrac{1}{a-b}-\dfrac{1}{a+b}} \div \dfrac{a}{b}$.

6. Solve $6(x-\frac{1}{2})(x+\frac{1}{3}) = 0$.

7. Simplify $\left\{\dfrac{a^{-2}}{b^{-2}}\sqrt{\dfrac{a}{b}}\right\}^{\frac{1}{2}}$.

75.

1. Find the value of

$$\frac{a+\sqrt{a^2+b^2}}{a^3-2b(a^2-b^2)} \text{ when } a=-4 \text{ and } b=-3.$$

2. Bracket together the equivalent expressions in the following list:

$$\frac{1}{a^3},\ a^2,\ a^{\frac{1}{2}},\ a^{-3},\ \sqrt{a},\ \sqrt{a^4},\ a^{\frac{4}{2}},\ \sqrt[6]{a^3},\ \frac{a^4}{a^7},\ a^3\times a^{-1}.$$

3. Divide $x^4-\frac{1}{x^4}$ by $x-\frac{1}{x}$.

4. Solve $\left.\begin{array}{l}(x+5)(y+7)=(x+1)(y+9)+112\\ 2x+10=3y+1\end{array}\right\}.$

5. Simplify $b^3\sqrt[3]{8a^6b^3}+4a\sqrt[3]{a^3b^{12}}-\sqrt[3]{125a^6b^{12}}$.

6. A traveller walks a certain distance at a certain rate. If he had gone $\frac{1}{2}$ a mile an hour faster, he would have walked the distance in $\frac{4}{5}$ of the time. If he had gone $\frac{1}{2}$ a mile slower, he would have been $2\frac{1}{2}$ hours longer on the road. Find the distance, and the rate at which he travelled.

7. A man walking $\frac{1}{3}$ of a mile above his ordinary rate gains $\frac{3}{4}$ of an hour in 39 miles. Find his ordinary rate of walking.

76.

1. Multiply $x^{\frac{3}{2}}-4x^{\frac{1}{2}}+2$ by $x-x^{\frac{1}{2}}$.

2. Solve $\frac{ax-1}{\sqrt{ax}+1}=4+\frac{\sqrt{ax}-1}{2}$.

3. Show that if $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}$, then $\frac{a}{b}=\frac{xa+yc+ze}{xb+yd+zf}$.

4. Divide $x^{\frac{1}{2}}y^{-\frac{1}{2}}$ by $x^{-\frac{1}{2}}y^{\frac{1}{2}}$.

5. Simplify $(3\sqrt{8})(2\sqrt{6})$.

6. Simplify $[\{a^2(ab^{\frac{1}{2}})^{\frac{2}{3}}\}^{\frac{1}{2}}]^3$.

7. Solve $x-15\frac{3}{4}+\frac{5}{x-15\frac{3}{4}}=6$.

77.

1. Two ships sail at the same time from the same port; one due north at the rate of 9 miles an hour, the other due east at the rate of 12 miles an hour. After how many hours will they be 60 miles apart?

2. Solve $\dfrac{a(a-b)}{x-a-b}+a+b-x=\dfrac{(b-a)b}{a+b-x}$.

3. Solve $\dfrac{a}{x}=\dfrac{x}{a-x}$.

4. Two bodies move toward each other, starting from points 1800 feet apart. The first body starts 5 seconds after the second body, and the two bodies meet halfway between the points. If the rate of the first body is 6 feet a second more than that of the second body, find the rate of each body.

5. Solve $9x^2-9x+2=0$.

78.

1. Multiply $a^2-a+1-a^{-1}$ by $a+1+a^{-1}+a^{-2}$.

2. If $\dfrac{a}{b}=\dfrac{c}{d}$ show that $\dfrac{a+b}{c+d}=\dfrac{a-b}{c-d}$.

3. What is the value of $\{(\frac{1}{64})^{\frac{2}{3}}\}^{-\frac{1}{2}}$?

4. Simplify $(a^{\frac{2}{3}}b^{-\frac{1}{2}})^{\frac{3}{4}}+(a^{-\frac{1}{2}}b^2)^{\frac{1}{3}}$.

5. Divide \$560 between A and B, so that for every dollar A receives B shall receive \$2.50.

6. Solve $\left.\begin{aligned} x^2-xy&=10 \\ xy-y^2&=\ \ 6 \end{aligned}\right\}$.

7. For what positive values of m and n does the expression $(-a)^{\frac{m}{n}}$ denote an imaginary quantity?

79.

1. Find the value of $\frac{x^2-xy+y^2}{x^2+xy+y^2}$ when
$$x=\frac{\sqrt{3}+1}{2} \text{ and } y=\frac{\sqrt{3}-1}{2}.$$

2. Find the L.C.M. of $a-1,\ 1-a^2,\ a-2,\ 4-a^2$.

3. Solve $a+x+\sqrt{2ax+x^2}=b$.

4. Find two numbers, the sum of which is 15, and the sum of their squares 113.

5. Two freight trains pass a station at an interval of 4 hours, proceeding at the rates of $11\frac{1}{2}$ and $17\frac{1}{2}$ miles an hour, respectively. At what distance from the station will the faster train overtake the other?

6. Solve $ax^2+bx+c=0$. What is the relation between a and c when the product of the roots of this equation is equal to unity?

80.

1. Two passengers have together 400 pounds of baggage. One pays \$1.20, the other \$1.80, for excess above the weight allowed. If all the baggage had belonged to one person he would have had to pay \$4.50. How much baggage is allowed free?

2. Find the fourteenth term of $(2a-b)^{16}$.

3. Find the square root of
$$16m^4+\tfrac{16}{3}m^2n+8m^2+\tfrac{4}{9}n^2+\tfrac{4}{3}n+1.$$

4. Find the H.C.F. of $x^4-ax^3+a^3x-a^4$ and x^3-a^3.

5. Find the simplest value of $\sqrt[3]{64x^3y^6z^9}+\sqrt[4]{x^4y^8z^{12}}$.

6. Solve $\frac{a}{2bx-b}-\frac{2bx+b}{ax^2-a}=\frac{1}{2x^2+x-1}+\frac{1}{2x^2-3x+1}$.

7. Find two numbers whose sum is 7, and the sum of their squares exceeds their product by 19.

81.

1. Find the value of $ax+by$ when

$$x=\frac{cq-br}{aq-bp} \text{ and } y=\frac{ar-cp}{aq-bp}.$$

2. Add $x^2-3xy-\frac{2}{3}y^2$, $2y^2-\frac{2}{3}y^3+z^2$, $xy-\frac{1}{3}y^2+y^3$, $2xy-\frac{1}{3}y^3$.

3. Solve $\dfrac{6x+13}{15}-\dfrac{3x+5}{5x-25}=\dfrac{2x}{5}.$

4. Solve $\sqrt{4x+9}-2\sqrt{x}=1.$

5. Solve $\left.\begin{aligned} x+1-\frac{3y+4x}{7}&=7-\frac{9y+33}{14} \\ y-3-\frac{5x-4y}{2}&=x-\frac{11y-19}{4}\end{aligned}\right\}.$

6. Find two numbers whose difference is 1, and the sum of their squares 313.

7. Find the L.C.M. of

$$x^2+5x+4,\ x^2+2x-8, \text{ and } x^2+7x+12.$$

82.

1. Solve $\dfrac{x^2}{2}-3x=\dfrac{x(a+x)}{2}-\left(\dfrac{ax}{2}+12\right).$

2. The difference between a number and its square is 18 more than the same difference would be if the number were 1 less. Find the number.

3. Find the L.C.M. of $a-b$, $a+b$, a^2-b^2, a^3-b^3, a^3+b^3.

4. Simplify $\dfrac{\dfrac{x^2+y^2}{y}-x}{\dfrac{1}{y}-\dfrac{1}{x}}\times\dfrac{x^2-y^2}{x^3+y^3}.$

5. Solve $ax^2+bx+c=0$, and state the conditions under which the values of x will be (i.) real, (ii.) equal, (iii.) imaginary.

6. Find the product of $2y^{\frac{1}{3}}\sqrt{2x^3}$, $\sqrt{6xy}$, and $\dfrac{x^{\frac{1}{3}}}{\sqrt{3y^{\frac{1}{3}}}}.$

7. Solve $x-\dfrac{1}{y}=y-\dfrac{1}{x}=\dfrac{3}{2}.$

83.

1. Find the H.C.F. of
$$x^4-2x^3+x^2-8x+8 \text{ and } 4x^3-12x^2+9x+1.$$
2. Divide $\dfrac{6\sqrt{b}}{25\sqrt[5]{a^3}}$ by $\dfrac{20c\sqrt[4]{b^3}}{21ab\sqrt[3]{a^2}}$.
3. A can do a piece of work in 10 days which B can do in 8 days; after A has been at work upon it 3 days, B comes to help him; in what time will they finish it?
4. Solve $\dfrac{x+3}{x-3}-\dfrac{x-3}{x+3}=a$.
5. What number is that to which if 1, 5, and 13, severally, be added, the first sum shall be to the second as the second to the third?
6. Expand $(a-3x)^6$.

84.

1. If $s=\frac{1}{2}(a+b+c)$, show that
$$s^2+(s-a)^2+(s-b)^2+(s-c)^2=a^2+b^2+c^2.$$
2. Subtract $\dfrac{1-3x}{1+3x}$ from $\dfrac{3x}{1-3x}$.
3. A man has 6 hours for an excursion. How far can he ride out in a carriage which travels 8 miles an hour so as to return home in time, walking back at the rate of four miles an hour?
4. Resolve into factors $x^8-\frac{1}{256}$.
5. Show that, if $\dfrac{a}{b}=\dfrac{c}{d}$, then $\dfrac{7a+5b}{8a-3b}=\dfrac{7c+5d}{8c-3d}$.
6. Solve $x-y=\frac{1}{6}$; $x^2-y^2=\frac{5}{36}$.
7. Solve $\dfrac{2b-x-2a}{bx}+\dfrac{4b-7a}{ax-bx}=\dfrac{x-4a}{ab-b^2}$.

85.

1. Show that the sum of the squares of any two different numbers is always greater than twice their product.

2. Reduce to its lowest terms $\dfrac{4+12x+9x^2}{2+13x+15x^2}$.

3. Find the value of x in

$$\left(\frac{10a^{\frac{2}{3}}}{3b^{\frac{5}{4}}}\right)^2 : x :: \sqrt{\frac{5a\sqrt[3]{a^2}}{4\sqrt[5]{a^2b^9}}} : \frac{9b^{-3}}{\sqrt{5}}.$$

4. Solve $\sqrt{x(3-x)} = \sqrt{x+1} + \sqrt{2(x-1)}$.

5. Extract the square root of $33 - 20\sqrt{2}$.

6. Expand $\left(1 - \dfrac{3y}{4}\right)^5$.

86.

1. Solve $\dfrac{a-2b-x}{a^2-4b^2} - \dfrac{5b-x}{ax+2bx} + \dfrac{2a-x-19b}{2bx-ax} = 0$.

2. Simplify $1 + \dfrac{x}{1+x+\dfrac{2x^2}{1-x}}$.

3. Solve $\dfrac{3x-1}{2x-1} - \dfrac{4x-2}{3x-2} = \dfrac{1}{6}$.

4. Expand $(2x-3y)^4$.

5. A vessel which has two pipes can be filled in 2 hours less time by one than by the other, and by both together in 2 hours 55 minutes. How long will it take each pipe alone to fill it?

87.

1. If $a=2$, $b=3$, $x=6$, $y=5$, what is the value of
$$a+2x-\{b+y-[a-x-(b-2y)]\}.$$

2. Simplify $\left\{\sqrt{3+4\sqrt{-1}}+\sqrt{3-4\sqrt{-1}}\right\}^2$.

3. Reduce to the lowest terms
$$\frac{2x^2-8x+6}{x^2-5x+4}\times\frac{x^2-9x+20}{x^2-10x+21}\div\frac{2x^2-10x}{x^2-7x}.$$

4. Solve $\dfrac{x}{x+b-a}+\dfrac{b}{x+b-c}=1$.

5. When are the hands of a clock together between the hours of 6 and 7?

6. The plate of a looking-glass is 18 inches by 12 inches, and it is to be framed with a frame of uniform width, whose area is to be equal to that of the glass. Find the width of the frame.

7. Show that $a^0=1$.

88.

1. Find the fourth term of $(2x-5y)^{12}$.

2. Divide $\dfrac{2}{x}-\dfrac{1}{a+x}+\dfrac{1}{a-x}$ by $\dfrac{a+x}{a-x}-\dfrac{a-x}{a+x}$.

3. Solve $(a-b)x=(a+b)y$; $x+y=c$.

4. A carriage, horse, and harness are worth, together, \$720. The carriage is worth $\frac{4}{5}$ of the value of the horse, and the harness $\frac{2}{5}$ of the difference between the values of the horse and carriage. Find the value of each.

5. Solve $\dfrac{x+13a+3b}{5a-3b-x}-1=\dfrac{a-2b}{x+2b}$.

89.

1. Solve $x^4 - 5x^2 + 4 = 0$.

2. Solve $\dfrac{\sqrt{2a^2 - x^2} + b\sqrt{2a - x}}{\sqrt{2a^2 - x^2} - b\sqrt{2a - x}} = \dfrac{\sqrt{a} + b}{\sqrt{a} - b}$.

3. Simplify $\dfrac{a^{-\frac{1}{2}}b^{\frac{1}{2}}}{\dfrac{1}{b^{\frac{1}{2}}c - \frac{1}{3}}} \times \dfrac{\dfrac{1}{b^{\frac{1}{3}}c^{\frac{1}{2}}}}{c^{\frac{1}{2}}a^{-\frac{1}{3}}} \times \dfrac{a^{\frac{1}{3}}c^3}{b^{\frac{1}{3}}c^{-\frac{2}{3}}}$.

4. Solve $\dfrac{x}{a} - \dfrac{y}{b} = 1,\ \ \dfrac{x}{b} + \dfrac{y}{a} = \dfrac{a}{b}$.

5. A hare is 50 of her leaps before a greyhound, and takes four leaps to his three. Two of the greyhound's leaps are equal to three of the hare's. How many leaps must the greyhound take to catch the hare?

90.

1. Extract the square root of $81a^{-\frac{5}{3}}b^{\frac{2}{5}}c$.

2. Express in the simplest form $\frac{1}{6}(\sqrt[3]{3} + \frac{1}{2}\sqrt[3]{192} + \sqrt[3]{81})$.

3. Simplify $\{(a^{-\frac{1}{2}})^{\frac{2}{3}}\}^{-\frac{3}{4}}$.

4. If $a : b = c : d = e : f$, show that
   $a : b = a + c + e : b + d + f$.

5. For building 108 rods of stone-wall, 6 days less would have been required if 3 rods more a day had been built. How many rods a day were built?

6. Solve $\left.\begin{aligned} x + y &= 5 \\ x^2 - 3xy + 4y^2 &= 7 \end{aligned}\right\}$.

91.

1. Simplify $\left(\dfrac{32a^{-10}x^{-\frac{15}{2}}}{b^{\frac{3}{4}}y^{\frac{5}{8}}}\right)^{-\frac{4}{5}}$.

2. A railway train, after travelling an hour, meets with an accident, which delays it 1 hour; after this it proceeds at $\frac{3}{5}$ of its former rate, and arrives 5 hours late. If the accident had occurred 50 miles farther on, the train would have been 3 hours 20 minutes late. Find the length of the line.

3. Prove that if $a : b = c : d$, then
$$a + b : a - b = c + d : c - d.$$

4. Solve $x^{-1} + ax^{-\frac{1}{2}} = 2a^2$.

5. Write the 5th term of $(3x^{\frac{1}{2}} - 4y^{\frac{1}{2}})^9$.

6. What is a surd? Give an example. What are similar surds? Give an example.

92.

1. Find the value of $\dfrac{x^3 - y^3}{x - y}$ when $x = 2^{\frac{1}{2}}a^{\frac{1}{2}}$ and $y = 2^{-\frac{1}{2}}a^{\frac{1}{2}}$.

2. Simplify $\dfrac{y}{2x^2 - 2xy} - \dfrac{4x + 5y}{4(x^2 - y^2)} + \dfrac{4x - 5y}{4(x - y)^2}$.

3. Simplify $\left(\dfrac{ay}{x^2}\right)^{\frac{1}{2}} \times \left(\dfrac{bx^2}{y^2}\right)^{\frac{1}{3}} \div \left(\dfrac{b^2a^3x^2}{y^3}\right)^{\frac{1}{6}}$.

4. Solve $\dfrac{\sqrt{x} + 2a}{\sqrt{x} + 2b} = \dfrac{\sqrt{x} + 4a}{\sqrt{x} + 3b}$.

5. Find x and y from the proportions
$$2x + y : y :: 3y : 2y - x;$$
$$2x + 1 : y :: 2x + 6 : y + 2.$$

6. A boat's crew row $3\frac{1}{2}$ miles down the river and back again in 1 hour and 40 minutes. If the current of the river is 2 miles per hour, determine their rate of rowing in still water.

93.

1. Add
$\sqrt{20a^2m-20acm+5c^2m}$ and $\sqrt{20c^2m-60acm+45a^2m}$.

2. Find the H.C.F. of
$4x^2y^2(x^2+xy+y^2)^2$, $6xy(x^6-y^6)$, and $18x^3(x^3-y^3)^2$.

3. Solve $abx^2+\dfrac{3a^2x}{c}=\dfrac{6a^2-ab-2b^2}{c^2}-\dfrac{b^2x}{c}$.

4. The circumference of the fore-wheel of a carriage is 9 feet, and that of the hind-wheel 12 feet. What is the distance gone over when the fore-wheel has made two more revolutions than the hind-wheel?

5. Which is the greater, $\sqrt{14}+\sqrt{7}$ or $\sqrt{19}+\sqrt{2}$? Prove your answer.

6. Solve $\left.\begin{aligned} x+y&=6 \\ x^3+y^3&=72 \end{aligned}\right\}$.

94.

1. Solve $\left.\begin{aligned} x^2+xy+2y^2&=44 \\ 2x^2-xy+y^2&=16 \end{aligned}\right\}$.

2. Find the two middle terms of $\left(\dfrac{x}{y}-\dfrac{y}{x}\right)^7$.

3. Solve $2x^2-21x+55=0$.

4. If x' and x'' are the roots of the equation $x^2-px+q=0$, show that $x'+x''=p$, and $x'x''=q$.

5. Form and solve the quadratic equation, the product of whose roots is 42 and their sum 13.

6. A number having two digits is to the number formed by interchanging the digits as 7 is to 4, and the sum of the two numbers is 132. Find them.

95.

1. For what value of m will the equation $2x^2+8x+m=0$ have equal roots?

2. Find the value of x^2+xy+y^2 if $x=1+\sqrt{2}$, $y=1-\sqrt{2}$.

3. A and B distribute \$100 in charity. A relieves 5 persons more than B, and B gives to each \$1 more than A. How many did they each relieve?

4. Solve $x^6-20x^3=189$.

5. Multiply $\left(\frac{ay}{x}\right)^{\frac{1}{2}}$, $\left(\frac{bx}{y^2}\right)^{\frac{1}{3}}$, and $\left(\frac{y^2}{a^2b^2}\right)^{\frac{1}{4}}$.

6. Find the H.C.F. of
$x^4+x^3-2x^2+3x-3$ and $x^5-4x^3-2x^2+3x+2$.

96.

1. Simplify $\dfrac{x^3-8y^3-z^3-6xyz}{x^3-x^2z-2x^2y}$.

2. Find the H.C.F. and the L.C.M. of
$34a^2bx^3y^4$, $27x^5y^3b^3$, $51y^2a^3x$.

3. Extract the square root of $x^4+\dfrac{1}{9}+2x+\dfrac{1}{x^2}-\dfrac{2x^2}{3}-\dfrac{2}{3x}$.

4. Solve $\left.\begin{array}{l} x^2-xy=\frac{3}{2} \\ xy+y^2=1 \end{array}\right\}$.

5. Solve $ax^2+2bx+c=0$, and show that the two roots are equal if b is a mean proportional between a and c.

6. Solve $\sqrt{x}+\sqrt{4+x}=\dfrac{4}{\sqrt{x}}$.

97.

1. Expand $\left(\sqrt{xy} - \frac{z}{2\sqrt{y}}\right)^5$.

2. Rationalize the denominator, and then find the approximate value of $\frac{7+2\sqrt{10}}{7-2\sqrt{10}}$.

3. Solve $15x - 3x^2 + 4(x^2 - 5x + 5)^{\frac{1}{2}} = -1$.

4. A man starts from the foot of a mountain to walk to its summit. His rate of walking during the second half of the distance is $\frac{1}{2}$ mile per hour less than his rate during the first half, and he reaches the summit in $5\frac{1}{2}$ hours. He descends in $3\frac{3}{4}$ hours, by walking 1 mile more per hour than during the first half of the ascent. Find the distance to the top, and the rates of walking.

98.

1. Divide $2\sqrt{3} + 3\sqrt{2} + \sqrt{30}$ by $3\sqrt{6}$.

2. Simplify $\frac{1}{4(1+\sqrt{x})} + \frac{1}{4(1-\sqrt{x})} + \frac{1}{2(1+x)}$.

3. Solve $\sqrt{x+3} + \sqrt{x+8} = 5\sqrt{x}$.

4. Find the mean proportional between $(a+b)^2$ and $(a-b)^2$.

5. Solve $\left.\begin{aligned} x - y &= 2 \\ 15(x^2 - y^2) &= 16xy \end{aligned}\right\}$.

6. Expand $(2\sqrt[5]{x^4} - \frac{1}{2}y^2)^4$.

99.

1. Simplify $2\sqrt[3]{40} + 3\sqrt[3]{108} + \sqrt[3]{500} - \sqrt[3]{320} - 2\sqrt[3]{1372}$.

2. Extract the square root of
$$1 + 4y^{-\frac{1}{3}} - 2y^{-\frac{2}{3}} - 4y^{-1} + 25y^{-\frac{4}{3}} - 24y^{-\frac{5}{3}} + 16y^{-2}.$$

3. Expand $\left(\frac{2x}{y^2} - \frac{1}{3}y\sqrt{x}\right)^4$.

4. Solve $\sqrt{2x+1} - \sqrt{x+4} = \frac{1}{3}\sqrt{x-3}$.

5. Solve $\frac{2x+1}{b} - \frac{1}{x}\left(\frac{1}{b} - \frac{2}{a}\right) = \frac{3x+1}{a}$.

6. The volume of a sphere varies as the cube of its diameter. Compare the volume of a sphere 6 inches in diameter with the sum of the volumes of three spheres whose diameters are 3, 4, 5 inches, respectively.

100.

1. Simplify $\frac{1}{a - \sqrt{a^2 - x^2}} - \frac{1}{a + \sqrt{a^2 - x^2}}$.

2. Rationalize the denominator of $\frac{8 - 5\sqrt{2}}{3 - 2\sqrt{2}}$, and find the approximate value of the fraction.

3. Form the quadratic equation whose roots are $a \pm \sqrt{-5b}$.

4. Solve $(x^{-\frac{1}{3}} + 2)(x^{-\frac{2}{3}} + 5) = x^{-1} + 8$.

5. A vessel which has two pipes can be filled in 2 hours less time by one than by the other, and by both together in 1 hour 52 minutes 30 seconds. How long will it take each pipe alone to fill the vessel?

101.

1. Multiply and free from negative exponents $-13a^{-1}c^{-3}$ by $-4a^{-3}b^{-6}c^{2}$.

2. Solve $\dfrac{x+\sqrt{9-x}}{x-\sqrt{9-x}}=\dfrac{7}{3}$.

3. Find an equivalent expression with a rational denominator for $\dfrac{\sqrt{21}+\sqrt{12}}{\sqrt{7}-\sqrt{3}}$.

4. Solve $\left.\begin{aligned} 2x^2+3y^2-xy&=31 \\ x-y&=\ \ 3\end{aligned}\right\}$.

5. Find the limit of the series $1+\frac{1}{2}+\frac{1}{4}\ldots\ldots$

6. Reduce to partial fractions by means of indeterminate coefficients $\dfrac{6x^2-4x-6}{(x-1)(x-2)(x-3)}$.

7. Expand $\left(1-\dfrac{x}{2}\right)^5$.

102.

1. Simplify and free from negative exponents $(-\frac{5}{2}a^{-2}b^{3}c^{-m}d^{-1})\times(\frac{2}{3}a^{2}b^{-3}c^{-2}d^{4})$.

2. Multiply $\sqrt[3]{5}-2\sqrt[3]{6}$ by $3\sqrt[3]{4}-\sqrt[3]{36}$.

3. Solve $\left.\begin{aligned} x^3-y^3&=63 \\ x-y&=\ \ 3\end{aligned}\right\}$.

4. How many terms of the series 54, 51, 48, etc., amount to 513? Explain the two answers.

5. Solve $3x^{\frac{3}{2}}-x^{-\frac{3}{2}}+2=0$.

6. Find the first five terms, and the $(r+1)$th term, in the expansion of $(1-x)^{-\frac{1}{2}}$.

103.

1. Simplify $\sqrt{\frac{a+b}{a-b}\sqrt{\frac{a+b}{a-b}}} \times \sqrt[3]{\frac{a-b}{a+b}\sqrt{\frac{a-b}{a+b}}}$.

2. Divide $x^{-6}+y^{-9}$ by $x^{-2}+y^{-3}$.

3. Solve $\frac{x+\sqrt{2-x^2}}{x-\sqrt{2-x^2}}=\frac{4}{3}$.

4. A debt can be discharged in a year by paying $1 the first week, $3 the second week, $5 the third week, etc. Required the amount of the debt, and the last payment.

5. Find the coefficient of x^{12} in the expansion of $(a^5-b^3x^2)^{\frac{5}{2}}$.

6. Solve $\left.\begin{aligned} x^2+xy+y^2&=52 \\ xy-x^2&=\ \ 8 \end{aligned}\right\}$.

104.

1. Simplify $\sqrt{5}\times\sqrt[3]{2}\times\sqrt[4]{4}$.

2. Find an equivalent expression with a rational denominator for $\frac{4+\sqrt{2}}{4+\sqrt{3}}$.

3. Solve $x^2-2x+6\sqrt{x^2-2x+5}=11$.

4. The height of a certain triangle is 4 inches less than the base. If the base be increased by 6 inches, and the height diminished by 6 inches, the area is diminished by $\frac{1}{8}$. Required the base of the triangle.

5. Find the fifth term of $(3x-2y)^{-10}$.

6. Resolve into partial fractions $\frac{x^2}{(x^2-1)(x-2)}$.

105.

1. Simplify the expression $\sqrt{\sqrt[3]{a+b}} \times \sqrt[3]{\sqrt{a-b}}$.

2. Solve $\dfrac{x}{m^2 p (x+a)} = \dfrac{x+a}{n^2 px}$.

3. Solve $x^2 = 21 + \sqrt{(x^2-9)}$.

4. Solve $\left.\begin{aligned} 5xy &= 84 - x^2y^2 \\ x - y &= 6 \end{aligned}\right\}$.

5. The sum of 11 terms of an arithmetical progression is 22, and the common difference is $\frac{2}{5}$. Find the first term.

6. Find the coefficient of x^9 in $(5a^3 - 4x^3)^7$.

106.

1. Simplify and free from negative exponents
   $(-7a^{-1}b^4c^{-5}) \times (3a^2b^{-5}c)$.

2. Find the square root of $28 + 5\sqrt{12}$.

3. Solve $x^2 - x + 5\sqrt{2x^2 - 5x + 6} = \frac{1}{2}(3x + 33)$.

4. Prove the formula for finding the sum of n terms of an arithmetical progression.

5. There are three numbers in arithmetical progression. If 1, 3, 9 are added to them, respectively, they are then in geometrical progression. Find the numbers.

6. Expand to five terms $\sqrt{1+2x}$.

7. Give an example of a series of terms in harmonical progression.

107.

1. Multiply $5\sqrt{14}+3\sqrt{5}$ by $7\sqrt{14}-2\sqrt{5}$.

2. Simplify $\sqrt[6]{(a^3 b \sqrt[5]{a^3} \times b \times c)^5}$.

3. Solve $x^{-\frac{1}{3}}+2=\dfrac{x^{-1}+8}{x^{-\frac{2}{3}}+5}$.

4. Solve $\left.\begin{aligned} 2x^2-3xy+y^2&=24 \\ 3x^2-5xy+2y^2&=33 \end{aligned}\right\}$.

5. How many terms of the series $19+17+15 \cdots\cdots$ amount to 91?

6. Prove the formula for finding the sum of n terms in geometrical progression.

7. Find the fifth term in the expansion of $\sqrt[3]{1+x}$.

108.

1. Divide $5-7\sqrt{3}$ by $1+\sqrt{3}$.

2. Find an arithmetical progression such that the second, third, fourth, and sixth terms may form a proportion.

3. Solve $\left.\begin{aligned} \frac{x}{y}-\frac{y}{x}&=\frac{15}{4} \\ x-y&=\frac{3}{2} \end{aligned}\right\}$.

4. A person bought a number of shares in a railroad, that cost him \$3000. He reserved 10 of the shares, and sold the remainder for \$2700, gaining \$4 a share. How many shares did he buy?

5. Find the limit of the sum of $9-6+4-\cdots\cdots$

6. Resolve into partial fractions $\dfrac{1}{x^4-a^4}$.

7. Find the fourth term in the expansion of $\left(5-\dfrac{x}{6}\right)^6$.

109.

1. Three numbers are as $6:11:20$. If 1 is added to each they are in geometrical progression. Find the numbers.

2. Solve $\dfrac{x+b}{a+b}+\dfrac{a}{b}=\dfrac{x^2+ab}{bx}$.

3. Divide 28 into two parts such that their squares shall be in the ratio $3:10$.

4. The first term of a series in arithmetical progression is 17, the last term $-11\frac{3}{8}$, and the sum $25\frac{7}{16}$. Find the common difference.

5. Find the limit of the sum of $1-\frac{2}{5}+\frac{4}{25}-\cdots\cdots$

6. Find the first five terms, and the $(r+1)$th term, of the expansion of $(1-x^2)^{-\frac{3}{2}}$.

110.

1. Simplify $\sqrt[4]{\left(\dfrac{a\sqrt{b}}{\sqrt[3]{ab}}\right)^3}$.

2. Find an equivalent expression with a rational denominator for $\dfrac{\sqrt{20}-\sqrt{8}}{\sqrt{5}+\sqrt{2}}$.

3. Solve $x+5=\sqrt{x+5}+6$.

4. A certain capital is invested at 4 per cent. If the number of dollars in the capital is multiplied by the number of dollars in the interest for 5 months, the result is $117{,}041\frac{2}{3}$. Required the capital.

5. The first and the ninth terms of a series in arithmetical progression are 5 and 22. Find the sum of twenty-one terms.

6. Find the middle term of $(1+x)^{2n}$.

7. Resolve into partial fractions $\dfrac{1}{x^2-9x+14}$.

111.

1. Simplify $\frac{b^2}{a^8} \times \frac{a^{-1}b^{-2}}{ab^{-3}} \times \frac{a^2b^{-1}}{b^4} \div \left(\frac{a^{-2}b^{-1}}{a^2}\right)^2$.

2. Find the H.C.F. of
$x^4 + 7x^3 + 14x^2 + 5x - 3$ and $2x^4 + 9x^3 + 8x^2 - x + 6$.

3. A merchant buys some goods for \$40, and sells them to another merchant, who, in his turn, sells them for \$48$\frac{1}{3}$. If each merchant makes a profit of the same rate per cent, determine what that rate is.

4. If $a : b = c : d = e : f$, show that
$\frac{a + 2c + 3e}{b + 2d + 3f} = \frac{2a + 3c + 4e}{2b + 3d + 4f}$.

5. The sum of five terms of an arithmetical series is 30, and the product of the first and last terms is 20. Form the series.

6. If α, β are the roots of the equation $x^2 - x + 1 = 0$, show that $\alpha + \beta = 1$, and $\alpha^2 + \beta^2 = -1$.

112.

1. Solve $x^2 + 3y^2 = 28$; $xy + y^2 = 12$.

2. In each of 3 battles 36 officers and 10 per cent of the men engaged are killed. At the end of the second battle the percentage of officers to men is two-thirds as great as at its commencement. The number of men at the end of the third battle is equal to the square of the number of officers at its commencement. How many officers and men were engaged in the first battle?

3. Find an arithmetical progression such that the sum of n terms shall be equal to n^2.

4. Solve $x^2 + \frac{5x}{2} - 2\sqrt{2x^2 + 5x + 3} = \frac{9}{2}$.

5. The terms of a ratio are 7 and 3; what number must be added to each in order that the ratio may be halved?

113.

1. Solve $x^2 + 2y^2 = 22$; $2xy + y^2 = 21$.
2. Simplify $2\sqrt{\frac{5}{3}} + \sqrt{60} - \sqrt{15} - \sqrt{\frac{3}{5}}$.
3. Solve $\sqrt{9x+40} + 2\sqrt{x+7} = \sqrt{x+44}$.
4. What is a ratio? Is it a quantity?
   If $m+n : m-n = x+y : x-y$, show that
   $x^2 + m^2 : x^2 - m^2 = y^2 + n^2 : y^2 - n^2$.
5. The thickness of a hollow cylinder varies directly as the amount of material, and inversely as the length of the cylinder and the sum of the radii of its internal and external surfaces. If the amount of material in a cylinder 50 feet long, whose radii of external and internal surfaces are 4 feet and 3 feet, respectively, be 1100 cubic feet, find the thickness of a cylinder 84 feet long, having the sum of its radii 5 feet, and containing 1650 cubic feet of iron.

114.

1. In an arithmetical progression, if s_1, s_2, s_3 denote the sums to the nth, $2n$th, $3n$th terms, respectively, prove that $s_3 = 3(s_2 - s_1)$.
2. In a geometrical progression, if l_1, l_2, l_3, denote the nth, $2n$th, $3n$th terms, respectively, prove that $l_2^2 = l_1 l_3$.
3. Reduce to its lowest terms $\frac{2x^3 - 11x^2y + 19xy^2 - 10y^3}{x^3 - 7x^2y + 13xy^2 - 6y^3}$.
4. Find the limit of $1 + \frac{1}{3} + \frac{1}{9} + \cdots$
5. A committee of 7 members is to be chosen out of a body of 20 protestants and 15 catholics in such a way that there shall be 3 protestants and 4 catholics. In how many different ways can such a committee be chosen.

115.

1. If A varies as B^2, B^3 as C^4, C^5 as D^6, and D^7 as E^4, show that $\frac{A}{E}\times\frac{B}{E}\times\frac{C}{E}\times\frac{D}{E}$ does not vary at all.

2. In how many ways can 2 white balls and 3 red ones be selected out of an urn containing 7 white balls and 10 red balls?

3. Solve $\frac{1}{\sqrt{x}-\sqrt{2-x}}-\frac{1}{\sqrt{x}+\sqrt{2-x}}=1.$

4. Insert 10 arithmetical means between 6 and 61, and find the sum of the whole series.

5. An express-train, travelling at uniform speed, after being an hour in motion, was delayed half an hour by an accident; after which it proceeded at three-fourths of its original rate of speed, and arrived at the end of its journey 1 hour and 50 minutes late. Had the accident occurred after the train had travelled a distance of 60 miles, it would have been 1 hour and 40 minutes late. Find the length of the line.

116.

1. Find an arithmetical progression such that the second, fourth, and eighth terms are in geometrical progression.

2. Solve $x^2=ax+by$; $y^2=ay+bx$.

3. If a be the greatest of the four proportionals a, b, c, d, show that $a-b>c-d$.

4. If four numbers be in proportion, and the first three be in arithmetical progression, show that the reciprocals of the last three are in arithmetical progression also.

5. In what scale of notation will 540 be the square of 23?

117.

1. If $\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}} = \sqrt{5}$, show that $\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}} = \pm 1$.

2. Prove that a ratio of greater inequality is diminished if the same quantity be added to both its terms.

3. Solve $4x^2 + xy = 6;\quad 3xy + y^2 = 10$.

4. The sum of 10 terms of a certain geometrical series is 33 times the sum of 5 terms of the same series. What is the common ratio?

5. Sum the series 3, $2\frac{7}{10}$, $2\frac{4}{10}$, ·····, to 21 terms.

6. The volume of a sphere varies as the cube of its radius, and that of a circular plate of given thickness as the square of its radius. If the volume of a sphere of radius 1 inch be equal to that of a plate of radius 2 inches, find the radius of a plate which is equal in volume to a sphere of radius 4 inches.

118.

1. In an arithmetical series the common difference is 2, and the square roots of the first, third, and sixth terms also form an arithmetical series. Find the series.

2. Find a geometrical progression such that the sum of an infinite number of terms shall be 4, and the second term shall be $\frac{3}{4}$.

3. What is the equation whose roots are double those of the equation $x^4 + x^2 - 6 = 0$?

4. A cattle dealer buys sheep, and sells them at a profit of 20 per cent. With the proceeds he again buys sheep, and sells them so as to gain 25 per cent. Once more he invests the proceeds in sheep, and this time he gains 16 per cent. If his last profit amounted to \$300, how much money did he invest at first?

5. Solve $x^3 - y^3 = a^3;\quad x - y = b$.

119.

1. Solve $(x+a)^5-(x-a)^5=242a^5$.

2. Solve $x^2-x+3\sqrt{x^2-3x+36}=2(x+26)$.

3. If the roots of the equation $3x^2-8x+5=0$ be α and β, show that the equation whose roots are $\frac{\beta}{\alpha}$ and $\frac{\alpha}{\beta}$ is $15x^2-34x+15=0$.

4. The sum of three quantities is y. The first varies inversely as x^2, the second inversely as x, and the third is constant. When $x=4$, 2, 1, then $y=3$, 7, 21, respectively. Find the equation between x and y.

5. Three numbers in geometrical progression, if multiplied by 3, 2, and 1, respectively, are in arithmetical progression. If the middle number is 18, what are the others?

120.

1. If $a:b=b:c$, then $\frac{b-c}{b}:\frac{a+b}{a}=\frac{a-b}{a}:\frac{b+c}{b}$.

2. If x and y are two numbers, A their arithmetical mean, and G their geometrical mean, then $x^2+y^2=2(A^2-G^2)$.

3. If z vary inversely as $3x+y$, and y vary inversely as x, and if, when $x=1$ and $y=2$, $z=3$, find the value of z when $x=2$.

4. Find two numbers such that their product, their sum, and three times their difference are in the proportion $5:2:4$.

5. The cost of an entertainment was \$120, which was to have been divided equally among the party; but four of them leave without paying, and the rest have each to pay \$2.50 extra, in consequence. Of how many did the party consist?

121.

1. If the number of visitors to a fair varies as the square of the number of degrees above 42° F., and if there are 1152 visitors when the temperature is 68°, how many visitors will there be if the temperature is 55°?

2. Insert between 6 and 16 two numbers such that the first three numbers may be in arithmetical progression, and the last three in geometrical progression.

3. If $\frac{1}{3}$ of the sum of the squares of the roots of the equation $ax^2 + bx + c = 0$ is equal to their product, find the relation between a, b, and c.

4. Solve $x + y = a;\quad x^2 + mxy + y^2 = b.$

5. What is the price of eggs per dozen when two more in a dollar's worth lowers the price one cent per dozen?

122.

1. Solve $x - y = a;\quad bx^2 - cx^2 = d.$
   What do the values of x and y become when $b = c$?

2. The side of a square is a. By joining the middle points of its sides another square is formed; by joining the middle points of the sides of this square a third square is formed. If the operation is continued indefinitely, find the limit of the sum of the areas of the squares.

3. Separate into partial fractions $\dfrac{6x^2 + 34x - 76}{(x-1)(x-3)(x-5)}$.

4. Find the fourth term of $\dfrac{1}{(a^2 - bx)^{\frac{2}{5}}}$.

5. How many different signals can be made with ten flags, of which three are white, two red, and the rest blue, if all are hoisted together, one above another?

123.

1. What number must be added to 20, 50, and 100, respectively, that the results may be in geometrical progression?

2. Find an arithmetical progression such that the second, fourth, and eighth terms are in geometrical progression.

3. The volume of a sphere varies as the cube of its radius. If three spheres, whose radii are 9 inches, 12 inches, 15 inches, respectively, are melted into one, what will be its radius?

4. Solve $x^2 - y^2 = x^2 + y^2 - xy = 3$.

5. In how many ways can a base-ball nine be arranged if three men can play in any position, and the others in any position except those of pitcher and catcher?

124.

1. What number must be added to 20, 50, and 100, respectively, that the results may be in harmonical progression?

2. The sum of four numbers in geometrical progression is 170, and the third exceeds the first by 30. Find them.

3. Solve $x^3 - 11x^2 + 37x - 35 = 0$, one root being 5.

4. Solve $4x + 4\sqrt{3x^2 - 7x + 3} = 3x(x - 1) + 6$.

5. Transform 3256 from the septenary to the duodenary scale.

6. Two steamers ply between the same two ports, a distance of 420 miles. One travels half a mile an hour faster than the other, and is two hours less on the journey. Find their rates.

125.

1. In an arithmetical series the second term is 21, the seventh term 41, the sum 1625. Find the number of terms.

2. The first and the seventh terms of a geometrical series are 2 and $\frac{1}{2}$. Find the intermediate terms.

3. Prove that the difference of the roots of the equation $x^2 - px + q = 0$ is equal to the difference of the roots of the equation $x^2 - 3px + 2p^2 + q = 0$.

4. A vessel is half full of a mixture of wine and water. If filled with water, the ratio of the quantity of water to the quantity of wine will be 9 times as great as if filled with wine. Determine the original quantities of water and wine.

5. Find the tenth term of $\dfrac{1}{(x+y)^m}$.

126.

1. If $a : b = c : d$ prove that
$$a(a + b + c + d) = (a + b)(a + c).$$

2. Solve $\dfrac{x + \sqrt{a^2 - x^2}}{x - \sqrt{a^2 - x^2}} = \dfrac{m}{n}$.

3. Solve $\left.\begin{aligned} x^3 + y^3 &= 225y \\ x^2 - y^2 &= 75 \end{aligned}\right\}$.

4. Insert 9 arithmetical means between 1 and -1.

5. There are four numbers, of which the first three are in geometrical progression and the last three in arithmetical progression. The sum of the first and last numbers is 16, and the sum of the second and third numbers is 12. Find the numbers.

6. Find the fortieth term of $(1 - x)^{-5}$.

127.

1. Find an equivalent expression with a rational denominator for $\dfrac{5\sqrt{2}+4\sqrt{3}}{3\sqrt{2}-2\sqrt{3}}$.
2. Solve $\dfrac{x^2}{y}+\dfrac{y^2}{x}=18$; $x+y=12$.
3. Two vessels, one of which sails faster than the other by 2 miles an hour, start at the same time on voyages of 1152 and 720 miles, respectively. The slower vessel reaches its destination one day before the other. What is the rate per hour of the faster vessel?
4. There are five numbers in arithmetical progression. Their sum is to the sum of their squares as 9 : 89. The sum of the first four numbers is 32. Find the numbers.
5. Find the limit of the sum of $4+3+\frac{9}{4}+\cdots$
6. Find the value of x in an infinite series in terms of y, if $y=1-2x+3x^2$.

128.

1. Reduce to the simplest form
   $2\sqrt[3]{3}(\sqrt[3]{9}-2\sqrt[3]{2\tfrac{2}{3}}+4\sqrt[3]{\tfrac{1}{3}}-3\sqrt[3]{2})$.
2. Solve $3x^{\frac{3}{2}}-x^{-\frac{3}{2}}+2=0$.
3. In t seconds a body falling freely describes $16t^2$ feet. The velocity of sound is 1100 feet a second, very nearly. If a stone dropped from the top of a tower is heard to strike the ground after 4 seconds, find the height of the tower.
4. The sum of three numbers in arithmetical progression is 24, and their product is 480. Find them.
5. The third and the seventh terms of a geometrical series are 12 and 192. Find the tenth term.
6. Find the tenth term of $\dfrac{1}{(1-5x)^2}$.
7. In what scale is 4954 expressed by 20305?

129.

1. Find the arithmetical mean, the geometric mean, and the harmonic mean between $a+b$ and $a-b$.

2. A man divided \$216 equally among a certain number of persons. If there had been three more persons, each would have received \$1 less. Required the number of persons.

3. Solve $x^4+y^4=706; \quad x-y=2.$

4. Find the value of the last term of an arithmetical series, having given the first term, the common difference, and the sum of the series.

5. Develop $(3a^{-1}-2x)^{-4}$ to five terms.

6. Separate into partial fractions $\dfrac{x^2-3x+2}{x^3-6x^2-x+30}$.

7. How many different arrangements can be made with the letters of the word *freshman*, if the letters *mna* are never separated?

130.

1. Divide $\frac{1}{2}\sqrt{\frac{1}{2}}$ by $\sqrt{2}+3\sqrt{\frac{1}{2}}$.

2. Solve $x+4+\left(\dfrac{x+4}{x-4}\right)^{\frac{1}{2}}=\dfrac{12}{x-4}$.

3. A sculptor purchased two cubical blocks of marble for \$2960, at \$5 per cubic foot. The length of the two together was 12 feet. Find the length of each.

4. An army on the march is advancing at the rate of 12 miles a day, when a detachment 55 miles in the rear is ordered to join it. How long will it take to do so, if it can advance 25 miles the first day, 24 the next, 23 the next, and so on?

5. The third term of a geometric series is 20, the eighth term is 640, and the sum of the series 20475. How many terms are there?

6. Insert five harmonic means between -1 and $\frac{1}{2}$.

7. What is the sixth term of $(a^2-x^2)^{-2}$?

131.

1. Solve $9x - 3x^2 + 4(x^2 - 3x + 5)^{\frac{1}{2}} = 11.$

2. Solve $x^2 + xy + y^2 = 52; \quad xy - x^2 = 8.$

3. From two towns, 168 miles apart, A and B set out to meet each other. A went 3 miles the first day, 5 the second, 7 the third, and so on. B went 4 miles the first day, 6 the second, 8 the third, and so on. In how many days did they meet?

4. The sum of three numbers in harmonical progression is 37, and the sum of their squares is 469. Find the numbers.

5. Expand to six terms, in a series of ascending powers of x, the fraction $\dfrac{1+2x}{1-x-x^2}$.

6. How many numbers between 10,000 and 15,000 can be formed with the digits 0, 1, 2, 3, 4, 5, 6, no repetitions being allowed, and each number to be divisible by 5?

132.

1. Rationalize the denominator of the fraction $\dfrac{\sqrt[3]{5}-\sqrt[3]{2}}{\sqrt[3]{5}+\sqrt[3]{2}}$.

2. Solve $x^{\frac{1}{4}} + 5x^{\frac{1}{2}} - 22 = 0.$

3. Solve $\dfrac{(x+y)^2}{a^2} + \dfrac{(x-y)^2}{b^2} = 8; \quad x^2 + y^2 = 2(a^2 + b^2).$

4. There are two arithmetical series which have the same common difference. The first terms are 3 and 5, respectively, and the sum of seven terms of one is to the sum of seven terms of the other as $2:3$. Determine the series.

5. What is the least integer which added to the ratio $9:23$ will make it greater than the ratio $7:11$?

6. Ten papers are to be set at an examination, four of them being in mathematics. In how many different orders can they be set so that the mathematical papers shall always come together?

133.

1. Solve $2x - y = 2$; $8x^3 - y^3 = 98$.

2. A and B start at the same time from the places C and D, A to travel from C to D, B to travel from D to C. When they meet, A had travelled thirty miles more than B. If they should continue at the same rates, A would finish his journey in 4 days and B in 9 days. Find the distance from C to D.

3. The first term of an arithmetical progression is $\frac{3}{2}$, and the difference between the third and the seventh terms is 3. Find the sum of n terms.

4. Find the limit of the sum of $\frac{2}{3} - \frac{1}{2} + \frac{3}{8} - \cdots\cdots$

5. Resolve into partial fractions $\dfrac{x^2}{x^3 - 7x^2 + 36}$.

6. From a society consisting of 12 men and 8 women, in how many ways can a committee of 4 men and 3 women be selected? In how many of these ways will a particular woman always be included?

134.

1. If a varies directly as $\sqrt{b}$, and inversely as c^3, and if $a = 3$ when $b = 256$ and $c = 2$, find b when $a = 24$ and $c = \frac{1}{2}$.

2. Solve $\sqrt[3]{x + 22} - \sqrt[3]{x + 3} = 1$.

3. Solve $x^2 + y^2 + x + y = 78$; $x + y + xy = 39$.

4. Find four numbers in arithmetical progression, such that the sum of the squares of the first and the second shall be 29, and the sum of the squares of the third and the fourth shall be 185.

5. Insert three geometrical means between $\frac{1}{2}$ and 128.

6. Find the sixth term of $(4a^2cx - 3c^{\frac{2}{3}}y^{\frac{1}{3}})^{\frac{1}{2}}$.

135.

1. Simplify $\sqrt{2\sqrt[3]{2\sqrt{2}}} \div \sqrt[3]{\sqrt{2\sqrt[3]{2}}}$.

2. Solve $x^{\frac{p}{n}} - ax^{\frac{p}{2n}} = b$.

3. A stone is dropped from the top of a tower, and when it has fallen exactly half-way to a window a feet below the top, another stone is dropped from the window. After how many seconds, reckoned from the time when the first stone is dropped, will the first stone overtake the second? The space described by a falling body in t seconds is $\frac{1}{2}gt^2$.

4. The difference between two numbers is 48, and the arithmetical mean exceeds the geometrical mean by 18. What are the numbers?

5. Find the value of x in an infinite series in terms of y, if $y = 1 + x - 2x^2 + x^3$.

6. A boat's crew consists of 8 men; 2 men can row only on the port side, and 2 others only on the starboard side. In how many ways can the crew be arranged?

136.

1. Solve $ay^2 + bxy - b = 0$; $bx^2 + axy - a = 0$.

2. Find in the simplest form the value of $(1 + \sqrt{-3})^6$.

3. If the second term of a geometric series is 24, and the fifth term 81, find the series.

4. If $a : b = c : d$, prove that $\sqrt{a+b} : \sqrt{b} = \sqrt{c+d} : \sqrt{d}$.

5. On how many nights may a different guard of 5 men be selected from a company of 36 men, and on how many of these would the oldest man have to serve?

6. If $y = x - \frac{x^2}{2} + \frac{x^3}{3} \cdots\cdots$, find the value of x in terms of y.

137.

1. Find the three cube roots of 1.

2. Solve $\sqrt{\frac{5x}{x+y}}+\sqrt{\frac{x+y}{5x}}=2;\quad xy-x-y=75.$

3. How many signals can be made with 3 blue and 2 white flags which can be displayed singly, or any number at a time, one above another?

4. Find the least number which, if divided by 15, leaves a remainder of 14, and if divided by 13, leaves a remainder of 12.

5. Find by the method of finite differences the sum of $3+11+31+69+131+\cdots\cdots$, to 20 terms.

6. How many balls are there in an incomplete square pile if the upper course consists of 529 balls and the base of 5184?

138.

1. Solve $x^2+y^2=axy;\quad x+y=bxy.$

2. From a horse-car station, a closed car leaves every 9 minutes, beginning at 7 o'clock A.M., and an open car every 16 minutes. At what times will an open car leave exactly 3 minutes after a closed car?

3. If on an average A solves 3 problems out of 5 which he tries, and B solves 2 out of 5, what is the probability that a problem which both try will be solved?

4. Find by the method of indeterminate coefficients the sum of $1+2^3+3^3+4^3+\cdots\cdots$, to twenty terms.

5. If $y=x+x^2+x^3$, find the value of x in terms of y, developing the series to six terms.

139.

1. Solve $\frac{1}{x}+\frac{1}{y}=\frac{1}{x+y}$; $\frac{1}{x^2}+\frac{1}{y^2}=\frac{1}{a^2}$.

2. A man wishes to make up as many *different* parties as he can out of 20 friends, each party consisting of the same number. How many should he invite at a time, and how many parties will there be?

3. If on an average 2 ships out of 5 return safe to port, what is the chance that out of 5 ships expected at least two will return?

4. A man wishes to build a wall 31 feet long, and has stones of 4 different lengths: viz., 2.2 feet, 2.5 feet, 3 feet, 4.1 feet. How many of each kind may he use in laying a course?

5. Solve by resolution into factors $x^4-3x^2+3x-1=0$.

6. Find by continued fractions the fourth convergent value for $\sqrt{10}$.

140.

1. What relation exists between the two roots of the equation $ax^2+bx+a=0$?

2. Solve $x^3-y^3=3$; $(x+y)(x^3+y^3)=7$.

3. Find by means of the binomial theorem the value of $\sqrt[4]{16.16}$ correct to five decimal places.

4. Find by the method of finite differences the sum of the infinite series $1+5x+9x^2+13x^3+17x^4+\cdots$

5. Expand to four terms by the method of indeterminate coefficients $\frac{2+x}{1+x+x^2}$.

6. Find the least integral values of x and y which will satisfy the equation $x^2-52y^2=1$.

141.

1. Define a *logarithm*, its *characteristic*, its *mantissa*. Prove that $\log mn = \log m + \log n$.

2. If 2 is the base of a system of logarithms, find the logarithms of 8, $\frac{1}{64}$, and $\sqrt[3]{16}$.

3. Given $\log a$, $\log b$, $\log c$, and $\log d$; how is the value of $\log \frac{a\sqrt[n]{c^m}}{b\sqrt{d}}$ found?

4. Find by logarithms the value of 24.13×6.052.

5. Find by logarithms the cube root of 3852.

6. Find the value of $\frac{8352 \times 3.69}{(30.57)^3}$.

142.

1. State and prove the rule for finding the logarithm of a quotient.

2. If the base of a system of logarithms is 3, find $\log 9$, $\log \frac{1}{81}$, $\log 27^4$, $\log \sqrt[7]{\frac{1}{243}}$.

3. What is colog 10 in the common system?

4. If $\log 6492 = 3.81238$, find $\log 0.00006492$.

5. Find the mean proportional between $(0.01)^{\frac{1}{5}}$ and $(0.2)^4$.

6. Find the value of $\frac{(0.005234)^{\frac{2}{7}} \times (0.017)^{\frac{1}{3}}}{24^{\frac{1}{3}}}$.

143.

1. What is the logarithm of 1 in any system? of the base of the system? of the reciprocal of the base? If the base is greater than 1, for what numbers are the logarithms positive, and for what negative? Give reasons for your answers.

2. If the base of a system of logarithms is $\frac{2}{5}$, find $\log \frac{125}{8}$, $\log \frac{625}{16}$, and $\log \sqrt[3]{\frac{4}{25}}$.

3. If $\log a = 0.78241$, and $\log b = 0.63575$, find $\log \sqrt{a^2 + b^2}$.

4. Find the value of $\dfrac{\sqrt[5]{0.35^4}}{\sqrt[3]{0.47^2}}$.

5. Solve $20^x = 100$.

144.

1. If the base of a system of logarithms is 64, find $\log 4$, $\log 16$, $\log 32$, $\log \frac{1}{256}$.

2. Of what number is $-\frac{1}{2}$ the logarithm in a system whose base is 16?

3. Find the value of $\dfrac{35^{\frac{1}{2}} \times 12^{1.7}}{13}$.

4. Find the square root of $\dfrac{\sqrt[3]{0.0125} \times \sqrt{31.15}}{0.00081}$.

5. Solve $10^{\frac{1}{x}} = 2.45$.

6. A man owes \$14,720.20. At the end of each year he pays \$2000. In how many years will the debt be paid, the rate of interest being 6 per cent?

145

1. If the base of a system of logarithms is $-\frac{2}{3}$, find $\log -\frac{8}{27}$, $\log \frac{4}{9}$, $\log \frac{81}{16}$.

2. Of what number is -5 the logarithm in the common system? in the system whose base is 3? in the system whose base is $\frac{1}{3}$?

3. For what base is $\log \frac{81}{10000}$ equal to 4?

4. Find the sixth root of 0.000000004096.

5. Find the value of $\dfrac{42 \times (0.0016)^{\frac{7}{4}}}{\sqrt[3]{108}}$.

6. Solve $(\frac{5}{3})^x = 17.4$.

146.

1. If the base of a system of logarithms is -6, find $\log 36$, $\log 1296$, $\log -\frac{1}{216}$.

2. Find the value of $(8.31)^{-0.27}$.

3. If $\sqrt[3]{8.37} : (0.84)^2 = \sqrt[5]{0.054321} : x$, find the value of x.

4. Find the value of $\dfrac{(15.6)^{\frac{1}{3}} \times (0.0045)^{\frac{3}{4}}}{(0.00065)^{\frac{1}{2}}}$.

5. Solve $(\frac{1}{2})^x = \frac{2}{3}$.

6. What sum will amount to \$1000 in 10 years at 5 per cent per annum compound interest?

147.

1. What is the logarithm of 512 to the base $2\sqrt{2}$?

2. What is the base of the system in which $\log 81 = -4$?

3. Find the value of $\dfrac{1.265 \times 0.01628}{2.2825 \times 64.28}$.

4. Find the mean proportional between $\sqrt{4.756}$ and $\sqrt[3]{(0.0078)^2}$.

5. Find the value of $\sqrt[4]{381 + \sqrt[3]{58}}$.

6. In how many years will a given sum of money treble itself, at 3 per cent per annum compound interest, payable semi-annually?

148.

1. What is the logarithm of $\frac{1}{144}$ to the base $2\sqrt{3}$?

2. Find the value of $\sqrt[3]{(24.4)^{-\frac{4}{5}}}$.

3. Given the Napierian base 2.71828; find the Napierian logarithms of 19, 23, 29, and 31, to five decimal places.

4. Find the value of $\left(\dfrac{5\sqrt[3]{138}}{\sqrt[5]{0.01}}\right)^{\frac{1}{2}}$.

5. A man places \$25,000 at 5 per cent compound interest, and draws out \$1000 at the end of each year. What will be due at the end of 12 years?

6. Solve $2^x = 769$.

149.

1. What is log 1 in any system? Why?

2. If the base of a system of logarithms is a, what is $\log a$, $\log\frac{1}{a}$, $\log a^3$, $\log\sqrt[5]{a^7}$, $\log\frac{a^m}{a^n}$, $\log\left(\frac{1}{a^n}\right)^{\frac{1}{m}}$?

3. Find the value of $\sqrt{\sqrt[3]{4}+\sqrt[3]{5}}$.

4. Find the value of $\sqrt[9]{\dfrac{347\times\sqrt[7]{0.0073}}{126\sqrt[3]{\frac{4}{9}}}}$.

5. Given the Napierian base $e = 2.71828$, $\log_e 5 = 1.60943$, $\log_e 11 = 2.39789$; find to five decimal places the common logarithms of 5 and 11.

150.

1. If the base of the system is a^m, what is $\log a^m$, $\log a^{5m}$, $\log\frac{1}{a^m}$, $\log 1$, $\log a^{mn}$, $\log\frac{a^m}{a^n}$?

2. Find the value of $\dfrac{\sqrt[3]{172500}}{\sqrt[5]{0.01}}$.

3. Find the value of $\dfrac{\sqrt[3]{69+3\sqrt[5]{1.19}}}{18\sqrt{0.95}}$.

4. Find the modulus for changing common logarithms to those whose base is 7.

5. Find to five decimal places log 54 in a system whose base is 12.

6. A spendthrift, at the age of 30, comes into possession of a fortune of \$200,000, which pays 5 per cent. He spends each year all his income, and also borrows \$2000 at 5 per cent compound interest. How old will he be when his fortune is all gone?

EXAMINATION PAPERS IN ALGEBRA.

I.

BOWDOIN COLLEGE, BRUNSWICK, ME.

Examination for Admission, June, 1883.

1. Divide $x^4 + x^3 + 5x - 4x^2 - 3$ by $x^2 - 2x - 3$.

   Divide $a^6 - b^6$ by $a^3 + 2a^2b + 2ab^2 + b^3$.

2. Resolve $x^{16} - y^{16}$ into five factors.

3. Find the L.C.M. of $(x + 2a)^3$, $(x - 2a)^3$, and $(x^2 - 4a^2)$.

4. From $\dfrac{a}{a-x}$ subtract $\dfrac{ax}{a^2 - x^2}$.

5. Multiply together $\dfrac{1-x^2}{1+y}$, $\dfrac{1-y^2}{x+x^2}$, and $1 + \dfrac{x}{1-x}$.

6. $(x + \frac{5}{2})(x - \frac{2}{3}) + \frac{3}{4} = (x+5)(x-3)$. Find x.

7. $\dfrac{4x+81}{10y-17} = 6$. $\dfrac{12x+97}{15y-17} = 4$. Find x and y.

8. Find the square root of $x^4 - x^3 + \dfrac{x^2}{4} + 4x - 2 + \dfrac{4}{x^2}$.

9. $\dfrac{1 + a + \sqrt{1-a^2}}{1 + a - \sqrt{1-a^2}}$. Reduce this to a fraction with rational denominator.

10. $\frac{1}{2}x^2 - \frac{1}{3}x + 7\frac{3}{8} = 8$. Find x.

    $x^6 - 8x^3 = 513$. Find x.

II.

Dartmouth College, Hanover, N.H.

Entrance Examination, June, 1883.

1. Divide $a^2 + \frac{1}{a^2} - 2$ by $\frac{1}{a} - a$.
2. Resolve $a^{12} - x^{12}$ into six factors.
3. Find the L.C.M. of $x^3 - x$, $x^2 - x - 2$, and $x^3 + 1$.
4. Reduce $\left(\frac{x-1}{x^2+y^2} - \frac{y^2x - x^3}{y^4 - x^4}\right)(y^2 + x^2)$ to its sinplest form.
5. Solve the equations: (i.) $2x - \left(x - \frac{x-1}{3}\right) = \frac{5x}{4}$;

$$\text{(ii.)} \begin{cases} \frac{x}{2} + \frac{y}{4} = 3 \\ \frac{2x}{3} - \frac{y}{6} = 2 \end{cases} \qquad \text{(iii.)} \begin{cases} \frac{1}{x} + \frac{1}{y} = \frac{1}{4} \\ \frac{1}{x} - \frac{1}{y} = \frac{1}{12} \end{cases}$$

6. Write the values of $8^{-\frac{2}{3}}$, 8^0, $16^{\frac{5}{4}}$, and $(8a^{\frac{1}{2}}b^2)^{\frac{2}{3}}$.
7. Multiply together $\sqrt[3]{abc^2}$, $a^{\frac{4}{3}}b^{-1}c^{\frac{1}{3}}$, and $a^{-\frac{2}{3}}b^{\frac{2}{3}}c^{-1}$.
8. Divide $a^3 - b^2$ by $a^{\frac{3}{4}} - \sqrt{b}$.

III.

Boston University, Boston, Mass.

Examination for Admission, June, 1882.

1. Multiply together $\frac{1-x^2}{1+y}$, $\frac{1-y^2}{x+x^2}$, and $\frac{1}{1-x}$; and simplify the result as much as possible.
2. Given $\frac{y-z}{2} - \frac{x+z}{4} = \frac{1}{2}$, $\frac{x-y}{5} - \frac{x-z}{6} = 0$, and $\frac{y+z}{4} = \frac{x+y}{2} - 4$. Find the values of x, y, and z.

3. Simplify $\frac{x}{x+1} - \frac{x}{x-1} + \frac{2x}{x^2-1}$.

4. Extract the square root of $40x + 25 - 14x^2 + 9x^4 - 24x^3$.

5. Given $3x^2 - 2xy = 15$; $2x + 3y = 12$. Find values of x and y.

6. Express $\frac{\sqrt[3]{5} \times \sqrt[4]{3}}{\sqrt{2}}$ with a single radical sign.

7. Expand $(1 - 2x^2)^5$ by the binomial theorem.

IV.

Brown University, Providence, R.I.

Examination for Admission, June, 1883.

1. Factor $x^4 - y^4$; also, factor $4a^4 - 8a^3x + 4a^2x^2$.

2. $\frac{6x+7}{15} - \frac{2x-2}{7x-6} = \frac{2x+1}{5}$. Find value of x.

3. A sum of money is divided equally among a certain number of persons; if there had been four more, each would have received a dollar less than he did; if there had been five fewer, each would have received two dollars more than he did; find the number of persons and what each received.

4. Multiply $a^{\frac{2}{3}} - a^{\frac{1}{3}} + 1 - a^{-\frac{1}{3}} + a^{-\frac{2}{3}}$ by $a^{\frac{1}{3}} + 1 - a^{-\frac{1}{3}}$.

5. $\sqrt{a+x} + \sqrt{a-x} = \sqrt{b}$. Find value of x.

6. $x + y = 4$; $\frac{1}{x} + \frac{1}{y} = 1$. Find values of x and y.

7. A boat's crew row $3\frac{1}{2}$ miles down a river and back again in 1 hour and 40 minutes; supposing the river to have a current of 2 miles per hour, find the rate at which the crew would row in still water.

8. Find sum of six terms of the geometrical progression of which $\frac{8}{5}$ is the first term and $\frac{8}{3}$ the second term.

V.

Mass. Institute of Technology, Boston, Mass.

Entrance Examination, June, 1882.

1. Factor the following expressions:
$$4x^2-12xy+9y^2;\ x^2+5x+6;\ x^3-8y^3.$$
2. Find the G.C.D. of
$$2x^3-4x^2-13x-7 \text{ and } 6x^3-11x^2-37x-20.$$
3. Find the L.C. M. of $4(1+x)$, $4(1-x)$, and $2(1-x^2)$.
4. Simplify $\frac{2a+b}{a-b}-\frac{2a-b}{a+b}-\frac{6ab}{a^2-b^2}$.
5. Multiply $a^n+a^{\frac{m}{2}}$ and $\sqrt{a}$ together.
6. Solve the equation $\frac{ax^2}{b-cx}+a+\frac{ax}{c}=0$.
7. Solve the simultaneous equations
$$\frac{x-4}{5}-\frac{y+2}{10}=0 \text{ and } \frac{x}{6}+\frac{y-2}{4}=3.$$
8. Extract the square root of $x^4-2x^3y+3x^2y^2-2xy^3+y^4$.
9. Solve the quadratic equation $x-\frac{14x-9}{8x-3}=\frac{x^2-3}{x+1}$.
10. Solve the simultaneous quadratic equations
$$\frac{1}{x}+\frac{1}{y}=5 \text{ and } \frac{1}{x^2}+\frac{1}{y^2}=13.$$

VI.

Entrance Examination, Sept., 1882.

1. Factor the following:
$$9m^2-24m+16;\ x^2-2xy+y^2-z^2.$$
2. Find the G.C.D. of
$$12x^3-9x^2+5x+2 \text{ and } 24x^2+10x+1.$$
3. Find the L.C.M. of x^2-1; x^2+2x-3; $6x^2-x-2$.

4. Simplify $\frac{3}{x-a}+\frac{4a}{(x-a)^2}-\frac{5a^2}{(x-a)^3}$.

5. Show that

$$(a+b\sqrt{-1})(a-b\sqrt{-1})=(a+b+\sqrt{2ab})(a+b-\sqrt{2ab}).$$

6. Solve the equation $\frac{x^2-a}{bx}-\frac{a-x}{b}=\frac{2x}{b}-\frac{a}{x}$.

7. Solve the simultaneous equations

$$\frac{x+y}{y-x}=\frac{15}{8};\ 9x-\frac{37+44}{7}=100.$$

8. Extract the square root of

$$x^6+3x^5+6x^4+7x^3+6x^2+3x+1.$$

9. Solve the quadratic equations

$$\frac{a}{3}+\frac{5x}{4}-\frac{x^2}{3a}=0 \text{ and } 19x^4+216x^7=x.$$

10. Solve the simultaneous quadratic equations

$$\frac{x}{a}+\frac{y}{b}=1, \text{ and } \frac{a}{x}+\frac{b}{y}=4.$$

VII.

Harvard College, Cambridge, Mass.

Examination for Admission, June, 1883.

1. Solve the equation $\frac{1}{x}=2-\frac{4ax^2-3b(x-2)}{2a(x^2+1)+3b}$.

2. A man walks, at a regular rate of speed, on a road which passes over a certain bridge, distant 21 miles from the point which the man has reached at noon. If his rate of speed were half a mile per hour greater than it is, the time at which he crosses the bridge would be an hour earlier than it is. Find his actual rate of speed, and the time at which he crosses the bridge. Explain the *negative* answer.

3. Find the prime factors of the coefficient of the 6th term of the 19th power of $(a-b)$. What are the exponents in the same term, and what is the sign?

4. Reduce the following fraction to its lowest terms:
$$\frac{x^4+2x^2+9}{x^4-4x^3+10x^2-12x+9}.$$

5. Prove that, if $a:b=c:d$, $\frac{a+b}{c+d}=\frac{a-b}{c-d}=\frac{a}{c}=\frac{b}{d}$.

6. Solve the equations, $xy=4-y^2$; $2x^2-y^2=17$. Find all the answers, and show what values of x and y belong together.

VIII.

Yale College, New Haven, Ct.

Examination for Admission, June, 1883.

1. Reduce the following expression to its simplest form:
$$\frac{1}{x(x-a)(x-b)}+\frac{1}{a(a-x)(a-b)}+\frac{1}{b(b-x)(b-a)}.$$

2. Resolve y^9-b^9 into three factors.

3. Change $xy^{-2}-2x^{\frac{1}{2}}y^{-1}z^{-\frac{1}{2}}+z^{-1}$ to an expression which will contain no negative exponents.

4. If $\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$, prove by the principles of proportion that $\frac{a}{b}=\frac{c}{d}$.

5. Find the value of $2a\sqrt{1+x^2}$ when $x=\frac{1}{2}\left(\sqrt{\frac{a}{b}}-\sqrt{\frac{b}{a}}\right)$.

6. Given $(7-4\sqrt{3})x^2+(2-\sqrt{3})x=2$, to find x.

7. The sum of two numbers is 16, and the sum of their reciprocals is $\frac{1}{3}$. What are the numbers?

8. Compute the value of the continued fraction

$$\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{4+\frac{1}{5}}}}.$$

9. Convert $\frac{1}{\sqrt{1+x^2}}$ into an infinite series by the method of indeterminate coefficients, or by the binomial theorem.

10. Insert three geometrical means between $\frac{1}{2}$ and 128.

IX.

Sheffield Scientific School, New Haven, Ct.

Entrance Examination, June, 1883.

Candidates for examination in this subject as a whole should take the whole of this paper; those for the first year's partial examination, the first part of it; those for the second year's partial examination, the second part.

State what text-book you have studied, and to what extent.

I.

1. Reduce to their simplest forms the fractions

(i.) $\frac{ac+bd+ad+bc}{af+2bx+2ax+bf}$; (ii.) $\frac{ax^m-bx^{m+1}}{a^2bx-b^3x^3}$.

2. Given $\frac{ace}{d}-\frac{(a+b)^2x}{a}-bx=ae-3bx$, to find x.

3. A sum of money, at simple interest, amounted in m years to a dollars, and in n years to b dollars. Find the sum and the rate of interest.

4. Prove that if $\frac{x-y}{m}<1-\frac{y}{x}$, and m is positive, then $x<y$.

5. (i.) Simplify $(a^2 b^3)^{\frac{1}{3}} + (a^2 c^6)^{\frac{1}{3}}$.

(ii.) Extract the square root of $6\,hm^{2n} + h^2 + 9\,m^{4n}$.

(iii.) Reduce $\dfrac{\sqrt{x+a}+\sqrt{x-a}}{\sqrt{x+a}-\sqrt{x-a}}$ to an equivalent fraction with a rational denominator.

II.

6. Given $15x^3 - 20x = 35$, to find x.

7. Given $\dfrac{x+\sqrt{x^2-9}}{x-\sqrt{x^2-9}} = (x-2)^2$, to find x.

8. Given $x^2 - xy = 48$ and $xy - y^2 = 12$, to find x and y.

9. The number of permutations of n things taken r together is equal to 10 times the number when taken $r-1$ together; and the number of combinations of n things taken r together is to the number when taken $r-1$ together as 5 to 3; required the value of n and r.

10. Expand $\dfrac{3+2x}{5+7x}$ into a series of ascending powers of x, by the method of indeterminate coefficients. (Four terms of the series will be sufficient.)

X.

Entrance Examination, Sept., 1883.

(State what text-book you have studied, and to what extent.)

1. Given $\dfrac{x-y+1}{x-y-1} = a$ and $\dfrac{x+y+1}{x+y-1} = b$, to find x and y.

2. Simplify (i.) $\sqrt{27} + 2\sqrt{48} + 3\sqrt{108}$.

(ii.) $(\sqrt[7]{a^2 b})^3 (\sqrt[7]{a^3 b^{12}})^4$.

(iii.) $\dfrac{x^{2p(q-1)} - y^{2q(p-1)}}{x^{p(q-1)} + y^{q(p-1)}}$.

3. Form an equation whose roots shall be 2 and -3. Resolve $x^2 - 3x + 4$ into two factors.

4. Given $\frac{1}{x} + \frac{1}{y} = 5$ and $\frac{1}{x^2} + \frac{1}{y^2} = 13$, to find x and y.

5. Given $\frac{3x + \sqrt{4x - x^2}}{3x - \sqrt{4x - x^2}} = 2$, to find x.

6. To deduce a formula for the sum of a geometric progression in terms of the first term, the ratio, and the number of terms.

7. Having 10 different letters, how many sets of two each can you form of them, differing by at least one letter?

8. Expand $\frac{1}{1 - 2x + x^2}$ into a series of ascending powers of x by the method of indeterminate coefficients. (Four terms of the series will suffice.)

9. Express $\log \sqrt[3]{\frac{ab^2c^4}{d^5}}$ in a form adapted to computation.

10. To deduce a formula for the amount of a given sum of money for a given time at a given rate of compound interest.

XI.

Amherst College, Amherst, Mass.

Examination for Admission, June, 1883.

1. Find the value of $6a - [4b - \{4a - (6a - 4b)\}]$.

2. Divide $a^{-3n} - b^{6n}$ by $a^{-n} - b^{2n}$.

3. Show that $a^0 = 1$; also, that $a^{-m} = 1 \div a^m$.

4. Resolve $a^{4m} - b^{4m}$ into its prime factors.

5. Find the G.C.D. of $a^4 - b^4$ and $a^3 + a^2 b - ab^2 - b^3$.

6. Given $3ax - 2bx - \frac{1}{3}c - \frac{1}{4}mx = \frac{2}{3}c + \frac{3}{4}mx - n - bx + 2ax$, to find x.

7. Divide the number a into two parts, such that the second part shall equal m times the first part plus n.

8. $3y - 2x = 9$; $7x + y = 26$; find x and y.

9. Multiply $\sqrt{a+c}$ by $\sqrt[3]{a+c}$.

10. $3x^2 - 4x = 15$. Find x.

11. Expand $(1 + x^2)^7$ by the binomial formula.

12. Find the $(2n)$th term of the series 1, 3, 5, 7, ·····

XII.

Williams College, Williamstown, Mass.

Entrance Examination, June, 1883.

1. Divide $x^2 + \frac{1}{x^2} + 2$ by $x + \frac{1}{x}$.

2. Add the fractions $\frac{a}{2a - 2b}$ and $\frac{b}{2b - 2a}$.

3. Simplify $\dfrac{a}{b + \dfrac{c}{d + \dfrac{e}{f}}}$.

4. The sum of two numbers is 5760, and their difference is equal to one-third of the greater. Find the numbers.

5. $\frac{x}{a} + \frac{y}{b} = 1$; $\frac{x}{3a} + \frac{y}{6b} = \frac{2}{3}$. Find x and y.

6. Solve the equation $\sqrt{3x+4} + \sqrt{3x-5} = 9$.

XIII.

Tufts College, College Hill, Mass.

Examination for Admission, June, 1881.

1. Divide $2a^{m+1}-2a^{n+1}-a^{m+n}+a^{2n}$ by $2a-a^n$.
2. Find the G.C.D. of $ab+am$, $bn+mn$, and b^2n-m^2n.
3. Simplify $\dfrac{x^4-\dfrac{1}{x^4}}{x+x^{-1}}$.
4. Solve $3x-\dfrac{3x-19}{2}-8=\dfrac{23-x}{4}+\dfrac{5x-38}{3}+10$.
5. Solve $\sqrt{4+x}=4-\sqrt{x}$.
6. Solve $\dfrac{1}{x}+\dfrac{2}{y}=\dfrac{11}{15}$ and $\dfrac{3}{x}+\dfrac{4}{y}=\dfrac{9}{5}$.
7. There are three numbers whose sum is 324; the second exceeds the first as much as the third exceeds the second; the first is to the third as five to seven. What are the numbers?
8. $\dfrac{x}{a}+\dfrac{b}{x}=c$. Find the values of x.

XIV.

Trinity College, Hartford, Ct.

Examination for Admission, June, 1883.

(One problem may be omitted in each of the three divisions indicated by the letters A, B, C.)

A.

1. Find the G.C.D. of $2x^2+x-1$, x^2+5x+4, and x^3+1.
2. Solve the equation $\dfrac{6x+7}{15}-\dfrac{2x-2}{7x-6}=\dfrac{2x-1}{5}$.

3. Two workmen together finish some work in 20 days; but if the first had worked twice as fast and the second half as fast, they would have done it in 15 days. How long would it take each alone to do the work?

B.

4. Multiply $2\sqrt{-3} - 3\sqrt{-2}$ by $4\sqrt{-3} + 6\sqrt{-2}$; divide $\sqrt{-5}$ by $\sqrt{-1}$. Explain the process in each case.

5. Solve the equation $\sqrt{x-3} - \sqrt{x-14} - \sqrt{4x-155} = 0$. Give and explain the rule for solving a quadratic equation.

6. Solve the equation $\dfrac{1}{x^2-1} + \dfrac{1}{3} = \dfrac{1}{3(x-1)} + \dfrac{1}{x+1}$; also, $x^4 + 4x^2 = 117$.

C.

7. Find two numbers such that their product is 96, and the difference of their cubes is to the cube of their difference as 19 to 1.

8. In an arithmetical progression, $a = 3$, $l = 42\frac{2}{3}$, $d = 2\frac{1}{3}$; find n and s. Explain the rule for the sum of a geometrical progression.

9. Expand $(a-b)^{11}$ and $\left(\dfrac{x}{2} + 3y\right)^5$ by the binomial theorem.

XV.

Wesleyan University, Middletown, Ct.

Examination for Admission, June, 1883.

1. When $a = 1$, $b = 0$, $c = 2$, find the value of
$$(3a+2)(2a+b) - a\{2c - a[(3a^2+6)+c]\}.$$
Multiply $a^{\frac{3}{2}}x^{\frac{3}{4}} + 2a^{\frac{1}{2}}x^{\frac{1}{4}}$ by $2a^2x^{\frac{1}{3}} - a^{\frac{1}{4}}x^2$.

2. Factor $y^3 - 64$; $a^2b^2 - 3ab - 4$.

3. Given $7\left(\frac{x}{2} - 1\right) - 3(2y + 8) = 0$; $\frac{2x+2}{9} - \frac{10y+9}{2} = 0$; to find x and y.

4. Solve at least two of the following:
   (i.) $6x^2 - 13x + 6 = 0$;
   (ii.) $3y^m\sqrt[3]{y^m} + \frac{2y^m}{\sqrt[3]{y^m}} = 16$;
   (iii.) $x^2 + y^2 = 8$; $\frac{1}{x^2} + \frac{1}{y^2} = \frac{1}{2}$.

   How do you "complete the square"?

5. From $a\sqrt{\frac{b^2z + b^2}{z - 1}}$ take $b\sqrt{\frac{a^2z - a^2}{z + 1}}$, and express the result in its simplest form.

6. Write the repeating decimal $0.\dot{3}$ as the sum of a geometrical progression. Find the limit of the sum.

XVI.

Cornell University, Ithaca, N.Y.

Entrance Examination, June, 1882.—*Elementary Algebra.*

1. Define: known and unknown quantities, positive and negative quantities, addition, a common multiple of two or more numbers, a radical, an equation, a theorem.

2. Resolve $m^4 - n^4$ into three prime factors.

3. Reduce the fraction $\frac{\sqrt{x^2 + xy + y^2}}{\sqrt{x - y}}$ to an equivalent fraction having a rational denominator.

4. Divide $x + y + z - 3\sqrt[3]{xyz}$ by $x^{\frac{1}{3}} + y^{\frac{1}{3}} + z^{\frac{1}{3}}$.

5. For \$8 I can buy 2 pounds of tea, 10 pounds of coffee, and 20 pounds of sugar, or 3 pounds of tea, 5 pounds of coffee, and 30 pounds of sugar, or 5 pounds of tea, 5 pounds of coffee, and 10 pounds of sugar. What are the prices?

6. Solve the equation $\frac{ax-b}{4}+\frac{a}{3}=\frac{bx}{2}-\frac{bx-a}{3}$.

7. Solve the equation $x+5+\sqrt{x+5}=6$, giving all the roots.

8. Solve the equation $\frac{x+a}{x-2a}+\frac{x-2a}{x+a}=1$, and get the sum and the product of the two roots.

XVII.

Entrance Examination, June, 1882.—Advanced Algebra.

1. Prove the formula for the development of $(a+x)^n$; and from this formula get the development of $(1+x+x^2)^3$, and four terms of $(a^2-x^2)^{-\frac{3}{2}}$.

2. Prove the formula for the sum of a geometrical progression, the first term, the ratio, and the number of terms being given. From this formula obtain an expression for the amount of a deferred annuity at compound interest; the annual payment, rate, and time being given.

3. By the method of differences, find log 24, by continuing the series: $\log 20 = 1.3010$, $\log 21 = 1.3222$,
$\log 22 = 1.3424$, $\log 23 = 1.3617$.
From the same data find $\log 21\frac{1}{2}$ by interpolation.

4. Prove that $\log_a b \times \log_b x = \log_a x$.

5. By the method of undetermined coefficients prove that, if $y = x + x^2 + x^3 + \cdots\cdots$, then also $x = y - y^2 + y^3 - \cdots\cdots$

6. By continued fractions, find five successive approximations to the value of $\sqrt{2}$.

7. Depress the equation $x^4 + 2x^3 + x^2 = 35x + 74$, by removing a commensurable root, and then find an incommensurable root correct to two decimal places.

8. Prove that, if the coefficients of an equation with one unknown quantity be real, any imaginary roots enter it in pairs.

XVIII.

Stevens Institute of Technology, Hoboken, N.J.

Specimen Entrance Examination Paper.

1. Multiply $\sqrt{2a}$ by $\sqrt[3]{3a}$.
2. Divide $1 - x$ by $1 + x$ to four terms.
3. Divide $\sqrt{3a}$ by $\sqrt[3]{2a^2}$.
4. Solve $1 - \dfrac{ax - b}{by - c} = 0$ and $a - \dfrac{2y - 3}{1 - x} = 1$.
5. Solve $a - \dfrac{\dfrac{3x - 2y}{b - 1}}{\dfrac{3y - x}{7}} = 2; \quad 4x - dy = 2$.
6. Solve $\sqrt{a - x} - \sqrt{b - x} = \sqrt{c - x}$.
7. Solve $3xy - 5y^2 = 2; \quad 5xy + 3x^2 = 1$.
8. Solve $\dfrac{x}{y} - \dfrac{y}{x} = a; \quad bx - \dfrac{cy - dx}{5} = ay$.
9. Establish the equation for the permutations of n things taken r at a time.
10. Prove that a proportion taken by *division* is a true proportion.
11. If x varies as $a + by$, and when $x = 1$, $y = 2$; and $x = 2$, $y = -5$; show that $7x = 9 - y$.
12. In an arithmetical series, given the common difference, first term, and number of terms, to find the sum and the last term.
13. In a geometrical progression, given the first term, number of terms, and the last term, to find the sum of the terms.

XIX.

Madison University, Hamilton, N.Y.

Entrance Examination, June, 1882.

1. Define: (i.) positive and negative quantities; (ii.) system of notation; (iii.) similar terms; (iv.) ratio; (v.) compound ratio; (vi.) proportion.

2. Multiply $a^{\frac{m}{n}}b^{-\frac{t}{s}} - a^{\frac{m}{2n}}b^{-\frac{t}{2s}} + 1$ by $a^{\frac{m}{2n}}b^{-\frac{t}{2s}} + 1$.

3. Rationalize the denominators of the following fractions:

$$\frac{1}{(2)^{\frac{1}{2}}},\ \frac{x^{\frac{1}{m}}}{y^{\frac{1}{n}}},\ \frac{3}{(3)^{\frac{1}{2}} + (5)^{\frac{1}{3}}}.$$

4. Given $\dfrac{3x^{\frac{1}{2}} - 4}{2 + x^{\frac{1}{2}}} = \dfrac{15 + (9x)^{\frac{1}{2}}}{40 + x^{\frac{1}{2}}}$, to find the value of x.

5. Four given numbers are represented by a, b, c, and d; what quantity added to each will make them proportional?

6. Suppose a body to move eternally in this manner: viz., 20 miles the first minute, 19 miles the second minute, $18\frac{1}{20}$ miles the third minute, and so on in geometrical progression; what is the utmost distance it can reach?

7. A hare, 50 of her leaps before a greyhound, takes 4 leaps to the greyhound's 3; but 2 of the greyhound's leaps are equal to 3 of the hare's. How many leaps must the greyhound take to catch the hare?

XX.

Vassar College, Poughkeepsie, N.Y.

Specimen Examination Paper for Admission.

1. Factor the following expressions:
   $9x^4 - y^2$; $m^2 - 2mn + n^2$; $n^5 - n$; $m^4 - n^4$; $x^3 + a^3$.

2. What is the rule for transposing a term from one side of an equation to the other; what is the principle?

3. Solve the equation $\frac{x}{a} - \frac{a}{a+b} = \frac{x}{a-b}$.

4. Find the fifth power of $2a^2$; the fourth root of $\frac{27a^{-3}b^{20}}{16d^{12}z^4}$.

5. Square $x - \frac{1}{x}$; cube $3 + \sqrt{2}$.

6. A and B engage to mow a field. A alone can mow it in b days, and B alone in c days. In what time can both together mow it?

7. Solve the equation $\sqrt{x+4} - \sqrt{x} = \sqrt{x + \frac{3}{2}}$.

8. Solve the the equations
$x^3 - y^3 = 215$; $x^2 + xy + y^2 = 43$.

9. Find four values of x in the equation $x^2 + \frac{1}{x^2} = a^2 + \frac{1}{a^2}$.

10. Form the equations whose roots are a, $-a$, and b.

11. Deduce the formula for the sum of n terms of a geometrical progression.

XXI.

College of New Jersey, Princeton, N.J.

Examination for Admission, Sept., 1883.

1. What text-book have you used?

2. Simplify the expression $\dfrac{\left(\dfrac{a^2+b^2}{b} - a\right)\dfrac{a^2-b^2}{a^3+b^3}}{\dfrac{1}{b} - \dfrac{1}{a}}$.

3. Solve the following equations:

$$5-6x+\frac{7x+14}{3}=\frac{17-3x}{5}-\frac{4x+2}{3};$$

$$\sqrt{x+x^{\frac{1}{2}}}=\frac{3}{2}\left(\frac{\sqrt{x}}{1+\sqrt{x}}\right)^{\frac{1}{2}}; \quad \frac{x+2}{x-2}-\frac{4-x}{2x}=\frac{7}{3};$$

$$x^{\frac{1}{3}}+x^{\frac{1}{6}}=6.$$

4. Solve the simultaneous equations:

$$\frac{x+3y}{2}=7\tfrac{1}{2}; \quad \frac{4x+5y}{4}-8.$$

5. Divide $x+y+z-3x^{\frac{1}{3}}y^{\frac{1}{3}}z^{\frac{1}{3}}$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}+z^{\frac{1}{3}}$.

6. Find the square root of

$$16x^4-16abx^2+16b^2x^2+4a^2b^2-8ab^3+4b^4.$$

XXII.

John C. Green School of Science, Princeton, N.J.

Entrance Examination, Sept., 1883.

1. What text-book have you used, and how much of it have you studied?

2. Solve the following equations:

$$(3a-x)(a+2x)=(5a+x)(a-2x);$$

$$1-\frac{x-5}{2x+1}=\frac{x-6}{x-2}; \quad (x^3-5)^2+29(x^3-5)=96.$$

3. Solve the following pairs of simultaneous equations:

$$\frac{3x}{5}-y=31; \quad x+\frac{y}{5}=33;$$

$$x^2+y^2=85; \quad xy=42.$$

4. Form the quadratic equation whose roots are $\frac{3}{2}$ and $\frac{2}{3}$.

5. Add $\sqrt[3]{\frac{1}{4}}$, $\sqrt[3]{\frac{1}{32}}$, and $\sqrt[3]{\frac{2}{3}}$.

6. Find the square root of $x^4+\dfrac{x^{-2}}{4}+4x^{\frac{2}{3}}-x+4x^{\frac{7}{3}}-2x^{-\frac{2}{3}}$.

XXIII.

UNIVERSITY OF PENNSYLVANIA, PHILADELPHIA, PA.

Examination for Admission, June, 1882.

1. Divide $x^6 - 9x^5 + 25x^4 - 3x^3 - 98x^2 + 156x - 72$ by $x^3 - 2x^2 - 4x + 8$ by synthetic division.
2. Simplify $\dfrac{a}{(a-b)(a-c)} + \dfrac{b}{(b-a)(b-c)} + \dfrac{c}{(c-a)(c-b)}$.
3. Simplify
$$2\sqrt[3]{189} - \sqrt[3]{9317} - 2\sqrt[3]{2401} + 2\sqrt[3]{448} - 2\sqrt[3]{56} + 3\sqrt[3]{875}.$$
4. Find the difference between $2 + 1 + \frac{1}{2} + \cdots\cdots$ to infinity, and $\frac{1}{6} + \frac{1}{3} + \frac{1}{2} + \cdots\cdots$ to 12 terms.
5. Find the square root of
$$x^8 - 12x^7 + 62x^6 - 180x^5 + 321x^4 - 360x^3 + 243x^2 - 96x + 16.$$
6. Find the value of x from $\sqrt{2x+6} + \sqrt{2x-5} = 11$.
7. Find x and y from
$$\frac{x}{x+2} + \frac{3}{y-3} = \frac{7}{12} \text{ and } \frac{2}{x+2} - \frac{3}{y-3} = -\frac{1}{12}.$$
8. Solve the equations $x + y - z = 4$, $2x - 3y + 4z = 20$, and $5x - 6y + 3z = 10$.
9. Find x from $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$.
10. Solve the equations $x^2 + 10xy = 11$ and $5xy - 3y^2 = 2$.

XXIV.

UNIVERSITY OF MICHIGAN, ANN ARBOR, MICH.

Examination for Admission, June, 1883.

1. Define *exponent*. Illustrate the significance of fractional and negative exponents. Write $\sqrt[3]{ax^2y^{-m}}$ without using the radical sign.

2. Define *elimination.* What method do you prefer, and why? Apply to the equations

$$\frac{a}{x}+\frac{b}{y}=m; \quad \frac{c}{x}+\frac{d}{y}=n.$$

3. Expand $(a^2+x^2)^{\frac{3}{2}}$ by the binomial formula.

4. Solve the equation $\dfrac{3\sqrt{x}-4}{2+\sqrt{x}}=\dfrac{15+\sqrt{9x}}{40+\sqrt{x}}.$

5. Produce the formulæ for *last term* and *sum* in arithmetical and geometrical progression.

6. Simplify $\dfrac{\left(1+\dfrac{y^2}{x^2}\right)^{\frac{2}{3}}}{x}$, $\dfrac{\left(1+\dfrac{y^4}{x^4}\right)^{\frac{3}{4}}}{x^2}.$

7. Rationalize the denominator of

$$\frac{\sqrt{x^2+x+1}+\sqrt{x^2+x-1}}{\sqrt{x^2+x+1}-\sqrt{x^2+x-1}}.$$

8. Solve the simultaneous equations

$$4(x+y)=3xy; \quad x+y+x^2+y^2=26.$$

9. Solve the simultaneous equations

$$x^2+xy+y^2=52; \quad xy-x^2=8.$$

10. Define *logarithm, mantissa, characteristic.* How can you extract roots by logarithms?

Given $\log x=\bar{2}.301030$, what is $\log x^{\frac{1}{3}}$?

XXV.

Lake Forest University, Lake Forest, Ill.

Entrance Examination, June, 1882.

1. Define *coefficient, exponent, similar quantities, monomial, binomial, equation.*

2. Find the G.C.D. of

$$x^4+2x^2+9 \text{ and } 7x^3-11x^2+15x+9.$$

3. Find the value of x in

$$\frac{3x-1}{7}+\frac{5-x}{4}-\frac{2x-4}{12}=2-\frac{x+2}{28}.$$

4. Explain the rule for subtraction, showing why the signs of the subtrahend are changed. Illustrate by diagram or numbers.

5. A man rows a boat with the tide 8 miles in 48 minutes, and returns against a tide two-fifths as strong in 80 minutes, what is the rate of the stronger tide?

6. The product of two numbers is 702, and their sum is 60. Find the numbers.

7. Factor x^2-2x-3 and $x^3-x^2-13x+24$.

8. Solve $\frac{\sqrt{a-x}}{x}-\frac{\sqrt{a-x}}{a}=\sqrt{x}$.

9. Solve $x+5\sqrt{37-x}=43$.

10. What number added to its reciprocal makes 2.9?

XXVI.

EDUCATION DEPARTMENT, ONTARIO.

Examination of Third Class Teachers, July, 1883.

1. Divide

(i.) $(a-b)c^3+(b-c)a^3+(c-a)b^3$ by $(a-b)(b-c)(c-a)$;

(ii.) $\frac{x^2+y^2}{x^3y^2}-\frac{x^2+y^2}{x^2y^3}$ by $\frac{1}{x}-\frac{1}{y}$.

2. What must be the values of a, b, and c, that x^3+ax^2+bx+c may have $x-1$, $x-2$, and $x-3$ all as factors?

3. Find the H.C.F. of

(i.) $3x^4-4x^3+1$ and $4x^4-5x^3-x^2+x+1$;

(ii.) $8x^3-y^3+27z^3+18xyz$ and $4x^2+12xz+9z^2-y^2$.

4. Simplify

(i.) $\left(\frac{4x^2}{y^2}-1\right)\left(\frac{2x}{2x-y}-1\right)+\left(\frac{8x^3}{y^3}-1\right)\left(\frac{4x^2+2xy}{4x^2+2xy+y^2}-1\right)$;

(ii.) $\frac{x^3+(a+b)x^2+(ab+1)x+b}{bx^3+(ab+1)x^2+(a+b)x+1}$.

5. Find the value of x that will make $\frac{ac+bd+ad+bc}{x-3c+2d}$ independent of c and d.

6. (i.) If $a+b+c=0$, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\left\{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right\}^2$.

(ii.) If $x=a^2+b^2+c^2$ and $y=ab+bc+ca$, then $x^3+2y^3-3xy^2=(a^3+b^3+c^3-3abc)^2$.

(iii.) If $2a=y+z$, $2b=z+x$, $2c=x+y$, express $(a+b+c)^3-2(a+b+c)(a^2+b^2+c^2)$ in terms of x, y, and z.

7. Find a value of a which will make the quantities

$$\frac{(a+b)(a+c)}{a+b+c} \text{ and } \frac{(a+c)(a+d)}{a+c+d}$$

equal to one another.

8. Solve the equations

(i.) $\sqrt{x+3}+\sqrt{x+2}=5$;

(ii.) $\frac{5-x}{3}+\frac{5-2x}{4}+\frac{x+1}{3}-\frac{2+5x}{2}=0$;

(iii.) $(x+a+b)(c+d)=(x+c+d)(a+b)$, where $c+d$ is not equal to $a+b$.

9. One side of a right-angled triangle exceeds the other by 3 feet, neither being the hypotenuse, and its area is 18 square feet. What are the sides?

10. A cistern with vertical sides is h feet deep. Water is carried away from it by one pipe $\frac{5}{6}$ as fast as it is supplied by another. Find at what point in the side the former pipe must be inserted that the cistern may fill in twice the time it would did water not flow from it at all.

XXVII.

UNIVERSITY OF TORONTO, TORONTO, ONT.

Junior Matriculation.—Annual Examination for Honors in Algebra, 1883.

1. Find the product of $(a+b)$, (a^2+ab+b^2), $(a-b)$, and (a^2-ab+b^2).

2. If a and b are positive integers, show that
$x^a \times x^b = x^{a+b}$.

3. Prove the rule for finding the G.C.M. of two quantities.
Find the G.C.M. of $6x^5+15x^4y-4x^3z^3-10x^2yz^2$ and $9x^3y-27x^2yz-6xyz^2+18yz^3$.

4. State the rule for extracting the square root of a compound quantity.
Extract the square root of x^2+y.

5. Solve the following equations:

   (i.) $3x+z=11,\ 2y+3z=16,\ 5x+4y=35$;

   (ii.) $\dfrac{x+a}{x-a}-\dfrac{x+b}{x-b}=c$;

   (iii.) $\dfrac{x}{a}+\dfrac{a}{x}=2+\dfrac{c}{x}$.

6. When are quantities said to be in geometrical progression, when in harmonical progression, and when in arithmetical progression? (i.) Find two harmonical means between a and b. (ii.) The first term of a geometric series is $\frac{1}{3}$, the ratio $\frac{1}{2}$, and the number of terms is 6; find the sum of the series.

7. Show that the number of combinations of n things taken r together is

$$\frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{1\times 2\times 3\ldots\ldots r}.$$

How many words of four letters can be formed out of the first 13 letters of the alphabet, having one vowel in each word?

8. Expand to five terms $(a+b)^{-\frac{1}{2}}$.

Show that $\left(\dfrac{1+x}{1-x}\right)^{\frac{1}{2}}$

$$=1+x+\tfrac{1}{2}(x^2+x^3)+\tfrac{3}{8}(x^4+x^5)+\tfrac{5}{16}(x^6+x^7)+\ldots\ldots$$

9. A number consists of two digits: when the number is divided by their sum the quotient is 4, and when divided by their difference the quotient is 12; find the number.

10. The crew of a boat rowed six miles down a river, and half-way back again, in 2 hours. Supposing the stream to have a current $2\frac{1}{2}$ miles an hour, find at what rate they would row in still water.

XXVIII.

College of Ottawa, Ottawa, Can.

Matriculation Examination. Session 1882–83.

1. Translate the following into common language:

$$\frac{\sqrt{5b}+3\sqrt{c}}{1+2a}=5x+\tfrac{1}{4}.$$

2. Divide (i.) $2a^2b+b^3+a^3+2ab^2$ by a^2+b^2+ab;
(ii.) $12n^{-2}y^{-4}$ by $-4xy^2$.

3. Find the prime factors of $25c^2d^4-9a^6c^4$.

Find the G.C.D. and L.C.M. of $9mx^2-6mx+m$ and $9nx^2-n$.

4. Find the sum of

$$\frac{a+b}{(b-c)(c-a)}+\frac{b+c}{(c-a)(a-b)}+\frac{c-a}{(a-b)(b-c)}.$$

Find the algebraic sum of

$$\frac{3}{1-2x}-\frac{7}{2x+1}-\frac{4-20x}{4x^2-1}.$$

5. Solve $x-\frac{3x-3}{5}+4=\frac{20-x}{2}-\frac{6x-8}{7}+\frac{4x-4}{5}.$

Find x, y, and z in the following equations:

$$x+y+z=24;\quad x-y+z=8;\quad x+y-z=6.$$

6. Raise $\frac{-bax^{-2}y^4}{5mn^{-1}}$ to the cube.

7. Give the simplest form of $\sqrt{3\sqrt[3]{3}}$.

8. Show why the square may be completed in the quadratic $3x^2-7x=20$, by the same rule as in $x^2+2x=24$, without introducing fractions.

9. Given $x^2-2xy=12$ and $x^2-y^2=12$, to find x and y.

10. A boy being asked how many sheep his father had, replied that $\frac{3}{2}$ of $\frac{1}{2}$ the flock would be 25 less than the whole flock. How many sheep had his father?

XXIX.

College of Ottawa, Ottawa, Can.

Matriculation Examination. Session 1883–84.

1. Clear away the parentheses, and reduce the following expression:

$$a+b-(2a-3b)-4(5a+7b)-(-13a+2b)$$
$$+3\{a-6(b-a)\}.$$

2. Give the three formulas for the expansion of $(a+b)^2$, $(a-b)^2$, and $(a+b)(a-b)$, and give an example for each formula.

3. Divide $5x-3-4x^2+x^4+x^3$ by $-3+x^2-2x$.

4. Find the G.C.D. and the L.C.M. of the three following expressions:
$(2x-4)(3x-6)$; $(x-3)(4x-8)$; $(2x-6)(5x-10)$.

5. Simplify $\dfrac{\dfrac{m^2+n^2}{n}-2m}{\dfrac{1}{n}+\dfrac{1}{m}} \times \dfrac{m^2-n^2}{m-n}$.

6. Solve the equations
$2x+4y-3z=22$; $4x-2y+5z=18$; $5x+7y-z=63$.

7. Extract the square root of
$15a^4b^2+a^6-6a^5b-20a^3b^3+b^6+15a^2b^4-6ab^5$.

8. Convert $\sqrt[3]{\frac{3}{4}}$ into such an expression, not a decimal, as shall not necessitate two extractions in finding the cube root of $\frac{3}{4}$.

9. Solve the following equation: $\frac{1}{2}x^2-\frac{1}{3}x+20\frac{1}{2}=42\frac{2}{3}$.

10. The hypotenuse of a right-angled triangle is 20 feet, and the area of the triangle is 96 square feet. Find the length of the legs.

11. Find the tenth term, and the sum of ten terms, of the series 1, 4, 10, 20, 35.

12. Develop $\dfrac{1+x}{1-x+x^2}$ into an infinite series by the method of undetermined coefficients.

13. Find the value of x in the equation $5^{\frac{2}{x}}=30$.

XXX.

McGILL UNIVERSITY, MONTREAL, CAN.

School Examination, June, 1883.

1. Multiply $1+2x-x^2-\frac{1}{2}x^3$ by itself, and find the value of the result if $1-2x=3$.

2. Find the remainder when $a^5-4a^3b^2-8a^2b^3-17ab^4-15b^5$ is divided by $a^2-2ab-3b^2$.

3. Simplify $\frac{2}{3}x(x+1)\{x+2-\frac{1}{2}(2x+1)\}$; $\dfrac{2(x^2-\frac{1}{4})}{2x+1}+\dfrac{1}{2}$.

4. Reduce the following fractions to their lowest terms:
$$\frac{a^2x+a^3}{ax^2-a^3};\ \frac{(x^4-a^4)(x-a)}{(x^2+a^2-2ax)(ax+x^2)};\ \frac{1+x^3}{1+2x+2x^2+x^3}.$$

5. Find the square root of
$$x^4+2x^3-x+\frac{1}{4} \text{ and of } \frac{4x^2-4x+1}{9x^2+6x+1}.$$

6. Solve the equations
$$\text{(i.) } 2x-\frac{x}{2}=18;\ \text{(ii.) } (m+n)(m-x)=m(n-x);$$
$$\text{(iii.) } 2x-\frac{y-3}{5}=4;\ \ 3y+\frac{x-2}{3}=9.$$

7. If ax^2+bx+c becomes 8, 22, 42, respectively, when x becomes 2, 3, 4, what will it become when $x=-\frac{1}{3}$?

8. Find two numbers which produce the same result, 7, whether one be subtracted from the other, or the latter be divided by the former.

9. In a certain school there are 6 boys to every 5 girls; if there were 2 boys less and 2 girls more, there would be the same number of each. Find the number.

10. Any odd number may be represented by $2v+1$. Prove that the difference of the squares of any two odd numbers is exactly divisible by 8.

XXXI.

University of Cambridge, Eng.

Second General Examination for the Ordinary B.A. Degree, Nov., 1880.
Time allowed, 3 hours.

1. Solve the equations (i.) $x-\frac{x-2}{3}=\frac{x+23}{4}-\frac{10+x}{5}$;

   (ii.) $\frac{a}{x-b}=\frac{b}{x-a}$; (iii.) $\frac{x}{9}+\frac{y}{8}=43,\ \frac{x}{8}+\frac{y}{9}=42$.

2. Solve the quadratic $ax^2+bx+c=0$, and determine the condition that its roots may be equal.

   If α, β be the roots, form an equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

3. Solve the equations

   (i.) $\frac{3x+2}{x-3}+\frac{3x-2}{x+3}=\frac{4x^2+12x+2}{x^2-9}$;

   (ii.) $\sqrt{x}+\sqrt{a+x}=\frac{a}{\sqrt{x}}$;

   (iii.) $(x+2y)(2x+y)=20,\ 4x(x+y)=16-y^2$.

4. The first term of an arithmetical progression of n terms is a, and the last term l. Find the sum, and also the common difference.

   If n be odd, and the sum of the even terms be subtracted from the sum of the odd, show that the result is $\frac{a+l}{2}$.

5. Find the sum of n terms of a series in geometrical progression.

   If the sum of a geometrical series to infinity be n times the first term, show that the ratio is $1-\frac{1}{n}$.

6. Find the sum of the series:

   (i.) $2+2\frac{1}{4}+2\frac{1}{2}+\cdots$, to 12 terms;

   (ii.) $\frac{1}{3}-\frac{1}{2}+\frac{3}{4}-\cdots$, to 8 terms;

   (iii.) $3\frac{3}{8}+2\frac{1}{4}+1\frac{1}{2}+\cdots$, to infinity.

7. Show that a ratio of greater inequality is increased by taking the same quantity from both its terms.

Show that the ratio $a-x:a+x$ is greater or less than the ratio $a^2-x^2:a^2+x^2$, according as the ratio $a:x$ is one of less or greater inequality.

8. Define proportion. When are quantities said to be in continued proportion?

If a, b, c, d be in continued proportion, show that $\left(\frac{a-b}{b-c}\right)^3=\frac{a}{d}$.

9. When is one quantity said to vary directly and when inversely as another?

The volume of a sphere varies as the cube of its radius: if three spheres of radii 9, 12, 15 inches be melted and formed into a single sphere, find its radius.

10. A and B start simultaneously from two towns to meet one another. A travels 2 miles per hour faster than B, and they meet in 7 hours; if B had travelled 1 mile per hour faster, and A at only half his previous pace, they would have met in 9 hours. Find the distance between the towns.

11. A wine-merchant buys spirit, and after mixing water with it, sells the mixture at two shillings per gallon more than he paid for the spirit, making $23\frac{3}{4}$ per cent on his outlay: if he had used double the quantity of water he would have made $37\frac{1}{2}$ per cent; what proportion of water was there in the mixture?

12. Two elevens, A and B, play a cricket match. A's first innings is the square of the difference of B's two innings, and A's second one-third the sum of B's two innings; A scored 60 more their first innings than in their second, and lost the match by one run. What were the respective scores, B having first innings?

XXXII.

UNIVERSITY OF CAMBRIDGE, ENG.

Second Previous Examination, Dec., 1880.—*Time allowed,* $2\frac{1}{2}$ *hours.*

1. Define *coefficient, term.*

Find the coefficient of x in the expression

$$\frac{x+a}{2}-\{2a-b(c-x)\}.$$

2. Find the continued product of

$x^2+3x+2,\ x^2-5x+6,\ x^2+2x-3,$

and multiply together

$x^2+(\sqrt{2}-1)x+1,\ x^2-(\sqrt{2}+1)x+1.$

3. Divide $x^4-(b-2)x^3-(2b-1)x^2-(b^2+2b-8)x+3b+3$ by $x^2+3x+b+1.$

4. Simplify (i.) $\dfrac{1}{x-2}+\dfrac{1}{x^2-3x+2}-\dfrac{2}{x^2-4x+3}$;

(ii.) $\dfrac{\dfrac{1+x}{1+x^2}-\dfrac{1+x^2}{1+x^3}}{\dfrac{1+x^2}{1+x^3}-\dfrac{1+x^3}{1+x^4}}.$

5. If a measures both b and c, prove that it will measure the sum of any multiples of b and c.

Find the G.C.M. of $1+x+x^3-x^5$ and $1-x^4-x^6+x^7$.

6. Solve the equations

(i.) $\dfrac{3x-5}{4}-\dfrac{x+1}{7}=2;$

(ii.) $\dfrac{1}{2(x+3)}=\dfrac{1}{3(x+2)}+\dfrac{1}{6(x+1)};$

(iii.) $\frac{1}{3}(x+y)=\frac{1}{5}(x-y),\ 3x+11y=4;$

(iv.) $3x^2+1=\dfrac{28x}{5};$

(v.) $\dfrac{x+a}{a+b}+\dfrac{b}{a}=\dfrac{x^2+ab}{ax};$

(vi.) $(x+1)(y+2)=10,\ xy=3.$

7. If $a+b=1$, prove that $(a^2-b^2)^2=a^3+b^3-ab$.

8. If $\frac{a}{b}=\frac{c}{d}$, prove that each of these fractions is equal to $\frac{a+mc}{b+md}$.

If $a+b$, $b+c$, $c+a$ are in continued proportion, prove that $b+c$, $c+a$, $c-a$, $a-b$ are proportionals.

9. When is one quantity said to vary as another?

If $\frac{1}{x}+\frac{1}{y}$ varies inversely as $x+y$, prove that x^2+y^2 varies as xy.

XXXIII.

University of Cambridge, Eng.

General Examination for the Ordinary B.A. Degree, June, 1881.
Time allowed, 3 *hours.*

1. Prove that $(x+4)^3-(x+1)^3=9(x+1)(x+4)+27$.

2. Simplify

(i.) $(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)-\frac{(b+c)(c+a)(a+b)}{abc}$;

(ii.) $\frac{a^2b^2-a^2-b^2+1}{ab-a-b+1}$.

3. Find the G.C.M. of x^3-4x^2+2x+3 and $2x^4-x^2-5x-3$.

4. Prove that, if m and n be positive integers, $(a^m)^n=(a^n)^m$.

Prove that $(\sqrt{3})^{3\sqrt{3}}=(3\sqrt{3})^{\sqrt{3}}$.

5. Solve the equations

(i.) $\frac{1}{3}(5x+1)+\frac{2x-3}{7}=x+\frac{8}{15}$;

(ii.) $\frac{1}{a+x}+\frac{1}{b+x}=\frac{a+b}{ab}$;

(iii.) $ax+by=2,\quad ab(x+y)=a+b$.

6. Show that the product of the roots of the equation $ax^2 + bx + c = 0$ is $\frac{c}{a}$.

Prove that the difference of the roots of the equation $x^2 + px + q = 0$ is equal to the difference of the roots of the equation $x^2 + 3px + 2p^2 + q = 0$.

7. Solve the equations

(i.) $(x-2)^2 + (x+5)^2 = (x+7)^2$;

(ii.) $ax^2 + 2bx = a - 2b$;

(iii.) $\sqrt{x+y} = \sqrt{y} + 2,\ x - y = 7$.

8. Find the sum of n terms of a G.P. of which the first and second terms are a and b.

If each term of a G.P. be squared, prove that the new series will also form a G.P.

9. If $\frac{a}{b} = \frac{c}{d}$, prove that each of these fractions is equal to $\frac{pa + qc}{pb + qd}$.

10. What is meant by saying that A varies as B?

If the volume of a cone varies jointly as its height, and the square of the radius of its base, show that if the heights of three cones of equal volume are in continued proportion, so also are the radii of their bases.

11. Find a fraction such that the denominator exceeds the square of half the numerator by unity, and the product of the sum and difference of the numerator and denominator is 64.

12. A vessel is half full of a mixture of wine and water. If filled up with water, the quantity of water bears to that of wine a ratio nine times what it would be were the vessel filled up with wine. Determine the original quantities of wine and water.

XXXIV.

University of Cambridge, Eng.

Previous Examination, June, 1881. — *Time allowed*, $2\frac{1}{2}$ *hours.*
Elementary Algebra.

1. Simplify $(x-a)^2-(x-b)^2-(a-b)(a+b-3x)$; and find the value of
$3(a-b)(a+2b)+2(a-2b)(2a-b)+2(2a-b)^2$, when $a=0$ and $b=-2$.

2. Divide $7a^3-22a^2b+4ab^2-3b^3$ by $a-3b$.

3. Resolve into factors
$x^2-2x-255$; $21x^2-13xy-20y^2$; $(x+2y)^3-y^3$.

4. Simplify

(i.) $\dfrac{x^2-16x-17}{x^2-22x+85}$;

(ii.) $\left(x-\dfrac{xy-y^2}{x+y}\right)\left(x-\dfrac{xy^2-y^3}{x^2+y^2}\right)\div\left(1-\dfrac{xy-y^2}{x^2}\right)$.

5. Find the L.C.M. of
(x^3-y^3), $(2x^2-3xy+y^2)$, and $(x^3+x^2y+xy^2)$.

6. Solve the equations

(i.) $\dfrac{x+1}{2}+\dfrac{x+4}{3}=\dfrac{10-x}{5}+5$;

(ii.) $2x(7x-10)=13(x-1)$;

(iii.) $7+\dfrac{5-x}{x-3}+\dfrac{4}{(x-1)(x-3)}=0$;

(iv.) $5abx+2y=16b,\ 3abx+4y=18b$;

(v.) $x^2+3xy=-8,\ y^2-xy=12$.

7. Extract the square root of
$9x^4-6x^3+43x^2-14x+49$.

8. Prove that $(a^m)^n=a^{mn}$.

Simplify $\dfrac{(a^{p-q})^{p+q}\times(a^q)^{q+r}}{(a^p)^{p-q}}$.

9. If α and β be the roots of the equation $x^2 - px + q = 0$, then will $p = \alpha + \beta$ and $q = \alpha\beta$.
Form the equation whose roots are 27 and -13.

10. When are three quantities said to be in continued proportion?
Show that if x, $(x+y)$, and $(x+2y+z)$ be in continued proportion, then x, y, z will also be in continued proportion.

11. Prove that if $x \propto y$ and $y \propto z$, then will $x \propto z$.
Given that x varies inversely as $(y^2 - 1)$, and is equal to 24 when $y = 10$; find x when $y = 5$.

XXXV.

University of Cambridge, Eng.

Previous Examination, June, 1881. — *Time allowed,* $2\frac{1}{2}$ *hours.*
Higher Algebra.

1. If the first two terms of an arithmetical progression are given, find the sum of the first n terms.
The sum of n terms of an arithmetical progression, whose first two terms are 43, 45, is equal to the sum of $2n$ terms of another progression, whose first two terms are 45, 43; find the value of n.

2. Find the sum of n terms of a geometrical progression whose first term and common ratio are given.
The sum of $2n$ terms of a geometrical progression, whose first term is a and common ratio r, is equal to the sum of n terms of a progression, whose first term is b and common ratio r^2; prove that b must be equal to the sum of the first two terms of the first series.

3. Sum the following series to 12 terms:

(i.) $1 - \frac{6}{5} - \frac{17}{5} - \cdots\cdots$

(ii.) $1 + \frac{5}{6} + \frac{25}{36} + \cdots\cdots$

(iii.) $1 - 1.2 + 1.44 - \cdots\cdots$

How many strokes are struck in a week by a clock that tells the hours?

4. If the sum of the first n terms of a series be $32n^2$, find the rth term.

5. For what values of m is $x^m + y^m$ divisible by $x + y$?

Divide $a^3 + 3a^2b + 3ab^2 + b^3 + c^3$ by $a + b + c$.

6. A clock gains 4 minutes per day; what time should it indicate at 6 o'clock in the morning in order that it may be right at 7.15 P.M. on the same day?

7. The first four nights of the boat-races both divisions rowed, and 32 bumps in all were made. The greatest number on one evening, in the first division, was reached twice, and was equal to the least number in the second division, which also occurred twice. This number is the middle one of five consecutive numbers, of which the first two represent the number of bumps the other two nights in the first division, and the last two represent the other bumps of the second division. How many bumps were made in the first division?

8. Define the logarithm of a number to a given base. Prove

$$\log_a \frac{m}{n} = \log_a m - \log_a n;$$

$$\log_a b \log_b a = 1.$$

9. Find the values of

$$\log_a a^m, \quad \log_{343} \sqrt[5]{49}, \quad \log_3 0.027.$$

10. Having given

$$\begin{aligned} \log 2 &= 0.3010300; \\ \log 3 &= 0.4771213; \\ \log 4.239 &= 0.627263; \\ \log 4.24 &= 0.627366; \end{aligned}$$

find the value of $\dfrac{2^{128}\, 3^{\frac{1}{5}}}{10^{35}}$.

XXXVI.

University of Cambridge, Eng.

Second General Examination for the Ordinary B.A. Degree, Nov., 1881. — *Time Allowed*, 3 *hours.*

1. Solve the equations

(i.) $\dfrac{x^3+3x^2+4x+2}{4x^2+13x+14}=\dfrac{x^2+x+1}{4x+5}$;

(ii.) $\dfrac{2(x+1)}{x^2-x+1}-\dfrac{2}{x+1}+\dfrac{1}{x+3}-\dfrac{1}{x-3}=0$;

(iii.) $x-11y=1,\ 111y-9x=99.$

2. Find two consecutive numbers, such that the fourth and eleventh parts of the less together exceed by 1 the fifth and ninth parts of the greater.

3. A certain number of two digits is multiplied by 4, and the product is less by 3 than the number formed by inverting its digits; if it be multiplied by 5, the tens' digit in the product is greater by 1, and the units' digit less by 2 than the units' digit in the original number: find the number.

4. Solve the equations

(i.) $3x^2-11x-4=0$;

(ii.) $\sqrt{7x+1}=3+\sqrt{2x-1}$;

(iii.) $x^2+xy-2y^2=-44,\ xy+3y^2=80.$

5. If the greater sides of a rectangle be diminished by 3 yards, and the less by 1 yard, its area is halved. If the greater be increased by 9, and the less diminished by 2, the area is unaltered; find the sides.

6. If the number of pence which a dozen apples cost is greater by 2 than twice the number of apples which can be bought for 1 *s.*, how many can be bought for 9 *s.*

7. Define *ratio*. If a be less than b, show that $a:b$ is a less ratio than $a+1:b+1$. What is the least integer which must be added to the terms of the ratio $9:23$, so as to make it greater than the ratio $7:11$?

8. The first and fourth terms of a proportion are 5 and 54; the sum of the second and third terms is 51; find them.

9. If A varies directly as P, inversely as Q, and directly as R, and, if when $P=a$, $Q=b$, $R=c$, $A=abc$, find A when $P=\frac{bc}{a}$, $Q=\frac{ca}{b}$, $R=\frac{ab}{c}$.

10. Find the sum of n terms of a geometrical progression, of which the first term is a and the common ratio r.

Sum:

$64+64\frac{1}{2}+\cdots\cdots$ to 29 terms in arithmetical progression;
$64+96\ +\cdots\cdots$ to 7 terms in geometrical progression.

11. The common difference of an arithmetical progression is 2, and the square roots of the first, third, and sixth terms are in arithmetical progression; find the series.

12. The sum of four numbers in geometrical progression is 170, and the third exceeds the first by 30; find them.

XXXVII.

University of Cambridge, Eng.

Second Previous Examination, Dec., 1881. — *Time Allowed*, $2\frac{1}{2}$ *hours.*

1. Simplify $6(a-2b)(b-2a)-(a-3b)(4b-a)-12ab$, and from the sum of $(2a-b)^2$ and $(a-2b)^2$ take the square of $2(a-b)$.

2. Define *multiplication*, *product*, and *coefficient*.
Divide $14a^4+15a^3b+33a^2b^2+36ab^3+28b^4$
by $7a^2-3ab+14b^2$.

3. Find the value of $(a-b)^2+(b-c)^2+(a-b)(b-c)+5c^2$ when $a=1$, $b=-2$, $c=\frac{1}{2}$.

4. Resolve into the simplest possible factors:
   (i.) $6x^2+5xy-6y^2$;
   (ii.) $x^3-13x^2y+42xy^2$;
   (iii.) $(a+2b+3c)^2-4(a+b-c)^2$;
   (iv.) $81x^4-625y^4$.

5. Define the highest common factor of two algebraical expressions.
Find the highest common factor of
$7x^3-10x^2-7x+10$ and $2x^3-x^2-2x+1$.

6. Reduce to simple fractions in their lowest terms:

(i.) $\dfrac{x^2-7xy+12y^2}{x^2+5xy+6y^2} \div \dfrac{x^2-5xy+4y^2}{x^2+xy-2y^2}$;

(ii.) $\dfrac{x-a}{x+a}+\dfrac{a^2+3ax}{a^2-x^2}+\dfrac{x+a}{x-a}$;

(iii.) $\dfrac{a-\dfrac{ab}{a+b}}{a^2+\dfrac{a^2b^2}{a^2-b^2}} \times \dfrac{\dfrac{1}{a^2}-\dfrac{1}{b^2}}{\dfrac{1}{a}-\dfrac{1}{b}}$.

7. Solve the equations

(i.) $\frac{x+1}{5}-\frac{2x-3}{9}=\frac{3x-2}{7}-10$;

(ii.) $\frac{x}{6}+\frac{x}{5}=14, \quad \frac{x}{9}+\frac{y}{2}=24$;

(iii.) $5x^2-17x+14=0$;

(iv.) $x^2+y^2=5a^2+5b^2+8ab, \quad xy=2a^2+2b^2+5ab$.

8. Find the value of $x^m \times x^n$, when m and n are positive integers.

Simplify $a^{2p+q} \times a^{p+4q} \div a^{q-p}$.

9. Define the antecedent and consequent of a ratio.

If $7(x-y)=3(x+y)$, what is the ratio of x to y?

10. Show that if $a:b::c:d::e:f$, then will
$a+3c+2e:a-c::b+3d+2f:b-f$.

11. Find two numbers such that their sum, their difference, and the sum of their squares are in the ratio $5:3:51$.

12. Prove that if x varies as $\frac{1}{y}+\frac{1}{z}$, and is equal to 3 when $y=1$ and $z=2$, then $xyz=2(y+z)$.

XXXVIII.

University of Cambridge, Eng.

General Examination for the Ordinary B.A. Degree, June, 1882. — *Time Allowed*, 3 *hours.*

1. Simplify $\dfrac{\left(\dfrac{a^2}{a^2+b^2}+\dfrac{b^2}{a^2-b^2}\right)(a^2+b^2)^2}{\dfrac{a}{a+b}+\dfrac{b}{a-b}}$.

2. Solve the equations

(i.) $\frac{3x+1}{4} - 2(6-x) = \frac{5x-4}{7} - \frac{x-2}{3}$;

(ii.) $27x^2 - 24x - 16 = 0$;

(iii.) $2y + \frac{3}{x} - 4 = 5y + \frac{12}{x} + 2 = y - \frac{2}{x} + 4.$

3. What is the meaning of the expressions x^{-n}, $x^{\frac{1}{p}}$?

Simplify $\frac{b^{xy} \times c^{yz+2y}}{b^{yz} \times c^{xy}} \times \left(\frac{b}{c}\right)^{y(z-x+1)}$.

4. Solve the equations

(i.) $x^2 - (a-b)x + (a-b+c)c = 2cx + ab$;

(ii.) $\sqrt{14x+9} + 2\sqrt{x+1} + \sqrt{3x+1} = 0$;

(iii.) $3x - 2\sqrt{xy} + 9 = 0,\ 5\sqrt{x} - 3\sqrt{y} - 3 = 0.$

5. Show that the sum of the roots of the equation $ax^2 - bx + c = 0$ is $\frac{b}{a}$.

If α and β are the roots of the above equation, form the equation whose roots are $-\frac{1}{\alpha}$, $-\frac{1}{\beta}$.

6. When is one quantity said to vary as another?

If $ax + by + 1 = 0$, where a and b are constant, and x and y are variable, and if the values of x are 2 and -9 when the values of y are 1 and -4, respectively, what will be the value of x when y is zero?

7. Define a geometrical progression.

Find the geometrical mean of $9x^2 - 12x + 4$ and $4x^2 - 12x + 9$.

8. The first term of an arithmetical progression is 38, and the fourth term is 86; find the sum of the first twelve terms.

The first term of a geometrical progression is 27, and the third term is 48; find the sum of the first six terms.

9. Find, to four places of decimals, the sum to infinity of the series $1 + \frac{1}{\sqrt{3}} + \frac{1}{3} + \cdots\cdots$

10. The perimeter of a rectangular field is 306 yards, and the diagonal is 117 yards. What is the area?

11. The expenses of a tram-car company are fixed, and when it only sells threepenny tickets for the whole journey it loses 10 per cent. It then divides the route into two parts, selling twopenny tickets for each part, thereby gaining 4 per cent, and selling 3300 tickets every week. How many persons used the cars weekly under the old system?

12. The price of a passenger's ticket on a French railway is proportional to the distance he travels; he is allowed 25 kilograms of luggage free, but on every kilogram beyond this amount he is charged a sum proportional to the distance he goes. If a journey of 200 miles with 50 kilograms of luggage cost 25 francs, and a journey of 150 miles with 35 kilograms cost $16\frac{1}{2}$ francs, what will a journey of 100 miles with 100 kilograms of luggage cost?

XXXIX.

University of Cambridge, Eng.

Previous Examination, June, 1882. —*Time Allowed,* $2\frac{1}{2}$ *hours.* — *Elementary Algebra.*

1. Find the value of
$$(3a-5b)(a-c)+c\{2a-c(3a-b)-b^2(a-c)\},$$
when $a=0,\ b=1,\ c=-\frac{1}{2}$.

2. Prove that
$$(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x).$$
Multiply $9a^{\frac{5}{6}}x^{\frac{1}{3}}+6a^{\frac{5}{12}}x^{\frac{5}{6}}+4x^{\frac{4}{3}}$ by $9a-6a^{\frac{7}{12}}x^{\frac{1}{2}}+4a^{\frac{1}{6}}x$.

3. Resolve into their simplest possible factors:

$$10x^2+x-2;\ x^2+4x-4y^2+4;\ a^3-3a^2b+3ab^2-b^3+c^3.$$

4. What is meant by a common multiple of two quantities? Prove that the sum of two quantities is a multiple of any of their common measures.

Find the L.C.M. of

$$x^3-3x^2y+3xy^2-2y^3,\ x^3-x^2y-xy^2-2y^3, \text{ and } x^4+x^2y^2+y^4.$$

5. Prove the rule for the multiplication of a fraction by an integer.

Reduce to fractions in their lowest terms:

(i.) $$\frac{1}{2a-3b+\dfrac{1}{2a-3b}-\dfrac{1}{2a-3b}};$$

(ii.) $$\left\{\frac{1}{x+y}-\frac{(x-y)^2}{x^3+y^3}\right\}\div\left\{\frac{x}{y}+\frac{y}{x}-1\right\}.$$

6. Prove that a quadratic equation cannot have more than two roots.

Solve the equations

(i.) $$4\left(x-\frac{3}{8}\right)-3\tfrac{1}{5}\left(\frac{x}{2}-1\right)=5\tfrac{9}{10};$$

(ii.) $$a(2x-y)+b(2x+y)=c(2x-y)+d(2x+y)=1;$$

(iii.) $$x^2-11x-42=0;$$

(iv.) $$\frac{1}{(x-1)(x-2)}+6=\frac{3}{x-2}+\frac{2}{x-1}.$$

7. A certain number, consisting of two digits, becomes 110 when the number obtained by reversing the digits is added to it; also the first number exceeds unity by five times the excess of the second number over unity. What is the number?

8. Define a third proportional to two quantities. Having given a third proportional to a and b, and also to b and a, determine a and b in terms of them.

If $a:b::c:d$, and x be a third proportional to a and c, and y to b and d, prove that the third proportional to x and y is equal to that to a and d.

9. When is a quantity said to vary inversely as another?

If a and b each vary inversely as c, prove that the sum of any given multiples of a and b varies inversely as any given multiple of c.

Given that $x-y$ varies inversely as $z+\frac{1}{z}$, and $x+y$ inversely as $z-\frac{1}{z}$; find the relation between x and z, provided that $x=1$ and $y=3$, when $z=\frac{1}{2}$.

XL.

University of Cambridge, Eng.

Previous Examination, June, 1882. — *Time Allowed,* $2\frac{1}{2}$ *hours.*
Higher Algebra.

1. A has twice as many pennies as shillings; B, who has 8*d.* more than A, has twice as many shillings as pennies; together they have one more penny than they have shillings. How much has each?

2. A man can walk a certain distance in 4 hours; if he were to increase his rate by one-fifteenth, he could walk one mile more in that time. What is his rate?

3. Solve the equations

   (i.) $\sqrt{5x+1}=2+\sqrt{x+1}$.

   (ii.) $x^2-2y^2=7,\ 2x+y=7$.

4. A man buys a number of articles for £1, and makes £1 1*s.* 0*d.* by selling all but two at 2*d.* apiece more than they cost. How many did he buy?

5. Find the sum of n terms of the progression
$a + ar + ar^2 + \cdots\cdots$

Find the sum of 10 terms of the progression
$64 + 96 + 144 + \cdots\cdots$

6. The fifth term of an arithmetical progression is 81, and the second term is 24; find the series.

7. Find an arithmetical progression whose first term is 3, such that its second, fourth, and eighth terms may be in geometrical progression.

8. If $a : x :: b : y :: c : z$, prove that
$$\frac{la^3 + mb^2y + ncz^2}{la^2x + mby^2 + nz^3} = \frac{pa + qb + rc}{px + qy + rz}.$$

9. Prove that $\log_a(pq) = \log_a p + \log_a q$.

10. Having given $\log_{10} 2 \doteq 0.3010300$, find the logarithm to base 10 of 25, 0.03125, and $(0.025)^{\frac{1}{3}}$.

XLI.

University of Oxford, Eng.

Local Examination, Junior Candidates, May, 1880. — *Time Allowed,* 2½ *hours.*

No credit will be given for any answer, the full working of which is not shown.

I. Algebra.

1. Find the value of $\dfrac{a^2 - c - 3ac(b - 2c)}{4c(a + b)} + \sqrt{2a - \dfrac{1+b}{c}}$, when $a = 1$, $b = 0$, and $c = -\frac{1}{2}$.

2. Multiply $x^4 - ax^3 + a^3x - a^4$ by $x^2 + ax + a^2$; also divide $p^4 - 9pq^3 + 18q^4$ by $p^2 - 3pq + 3q^2$.

3. Simplify

(i.) $\left(\frac{2x+y}{y}-\frac{y}{2x+y}\right)\left(\frac{x}{x+y}-\frac{x+y}{x}\right)$;

(ii.) $\frac{1}{x+1}-2\frac{x+2}{x^2-1}+\frac{x+3}{(x-y)^2}$.

4. Find the G.C.M. of x^3-6x-4 and $3x^3-8x+8$; also the L.C.M. of $(3a^2-3ab)^2$, $18(a^3b^2-ab^4)$, and $24(a^3b^3-b^6)$.

5. Solve the equations

(i.) $\frac{5x-3}{8}-\frac{1}{3}(x-2\frac{1}{2})=\frac{2x-1}{10}+1\frac{3}{4}$.

(ii.) $\frac{x+3}{y+4}=2\frac{x+1}{2y+7}$, $6(x+\frac{1}{2})=11(y+5)$.

II. Higher Algebra.

6. Solve the equations

(i.) $\frac{2x}{x+1}+\frac{10}{9}\frac{x+1}{x}=3$;

(ii.) $a(a-b)x^2+b(a+b)x-2b^2=0$;

(iii.) $x^2+2xy=\frac{3}{4}$, $x^2-4y^2=1\frac{1}{4}$.

7. The sum of 2 numbers is 35; and their difference exceeds one-fifth of the smaller number by 2; find the numbers.

8. After £12 have been divided equally among a certain number of men, an additional shilling apiece is given to them; and it is then found that each possesses as many shillings as there are men. Find the number of the men.

9. Prove that if b be a mean proportional between a and c, then $a^2+2b^2 : a :: b^2+2c^2 : c$.

10. Sum to 6 terms the series $\frac{1}{2}+1\frac{1}{4}+3\frac{1}{8}+\cdots\cdots$ Also insert 12 arithmetic means between $-\frac{1}{5}$ and 5.

XLII.

University of Oxford, Eng.

Local Examination, Senior Candidates, May, 1880. — *Time Allowed,* $2\frac{1}{2}$ *hours.*

No credit will be given for any answer, the full working of which is not shown.

Candidates are reminded that in order to pass in mathematics they must satisfy the Examiners in the first part of this paper.

I. Algebra to Quadratic Equations.

1. If $m = \dfrac{\sqrt{17}+1}{2}$ and $n = \dfrac{\sqrt{17}-1}{2}$, find the value of $m^2 + n^2$.

2. Find the G.C.M. of $2x^3+3x^2-7x-10$, $5-9x-4x^2+4x^3$; and the L.C.M. of a^2+ab, b^2+ab, ab.

3. Simplify $\dfrac{1}{x-1} - \dfrac{2}{x} + \dfrac{1}{x+1}$.

4. Find the square root of $4x^4 - 20x^3 + 13x^2 + 30x + 9$.

5. Multiply $x^{\frac{1}{2}} + 2x^{\frac{1}{4}} + 2$ by $x^{\frac{1}{2}} - 2x^{\frac{1}{4}} + 2$, and divide $a^2b^{-2} + b^2a^{-2} + 1$ by $ab^{-1} + ba^{-1} - 1$.

6. Solve the equations

   (i.) $\dfrac{1+x}{1-x} = \dfrac{21}{20}$;

   (ii.) $x = (2-x)(2+x)$;

   (iii.) $\dfrac{a}{x} + \dfrac{x}{a} = \dfrac{b}{x} + \dfrac{x}{b}$;

   (iv.) $x^2 + y^2 = 169 = 5x + 12y$.

7. Two rectangular fields each contain one acre; one of the fields is four poles shorter and two poles broader than the other. Find the length and breadth of each field.

II. Higher Algebra.

8. A spends £a in buying a number of articles, all at the same price; B spends £b in the same way, except that he buys n more articles than A buys, and pays £c less for each. Find an equation to determine the number of articles bought by A.

9. Solve the equations

(i.) $4^x + 1 = 5 \times 2^{x-1}$;

(ii.) $x^4 - 3x^3 - 2x^2 - 3x + 1 = 0$;

(iii.) $3x + 4y = 23$ (in positive integers).

10. Prove the formulæ for finding the nth term and the sum of n terms of an arithmetic progression, the first term and the common difference being known.

A man pays his gardener 15s. a week for the first fortnight; at the end of the first and of every succeeding fortnight he raises the wages 6d. per week. What will the gardener have received in all at the end of fifty weeks?

11. Find the cost of an annuity of £A per annum, to be paid quarterly, and to continue for p years; the first payment to be made at the end of the first quarter, reckoning compound interest, at the rate of £r per cent per annum, to be due at the date of each quarterly payment.

12. Enunciate the binomial theorem.

Show that the coefficient of the middle term in the expansion of $(1+x)^{2n}$ is the sum of the coefficients of the two middle terms in the expansion of $(1+x)^{2n-1}$.

13. Prove that

(i.) $\left(1 + \frac{1}{\lfloor 1} + \frac{1}{\lfloor 2} + \cdots\cdots\right)^x = 1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \cdots\cdots$;

(ii.) $\log_e \sqrt{\frac{1+x}{1-x}} = \frac{x}{1} + \frac{x^3}{3} + \frac{x^5}{5} + \cdots\cdots$

XLIII.

University of Oxford, Eng.

Local Examination, Junior Candidates, June, 1881. — *Time Allowed,* $2\frac{1}{2}$ *hours.*

No credit will be given for any answer, the full working of which is not shown.

I. Algebra.

1. Find the value of $a^3+b^3-c^3+3abc$:

(i.) when $a=\frac{1}{2},\ b=\frac{1}{3},\ c=\frac{1}{3}$;

(ii.) when $c=a+b$.

2. Multiply together $x^2-7x+6,\ \ x^2+7x-18,\ \ x^3-1$, and express the result in simple factors.

3. Find the G.C.M. of $2x^3+7x^2+10x+5$ and x^3+3x^2+4x+2, and the L.C.M. of $6xy^2(x+y), 3x^3(x-y)^2$, and $4(x^2-y^2)$.

4. Simplify

(i.) $\left(\dfrac{x+y}{y(x^2+y^2)}-\dfrac{x+y}{x(x^2+y^2)}+\dfrac{1}{xy}\right)\left(\dfrac{1}{y^2}+\dfrac{1}{x^2}\right)$;

(ii.) $\left(1-\dfrac{y^4}{x^4}\right)\div\left(\dfrac{x}{y}+\dfrac{y}{x}\right)$.

5. Solve the equations

(i.) $\dfrac{\frac{1}{3}-x}{2}+\dfrac{1-\dfrac{x}{2}}{\frac{3}{2}}=\dfrac{x}{4}+\dfrac{x-1}{3}$;

(ii.) $9x-8y=1,\ 12x-10y=1$.

6. Into a cistern one-third full of water 31 gallons are poured, and the cistern is then found to be half full; find its capacity.

II. HIGHER ALGEBRA.

7. Solve the equations

(i.) $x = \frac{55}{64} + \frac{x^2}{49}$;

(ii.) $ax^2 - \frac{6c^2}{a+b} = cx - bx^2$;

(iii.) $\frac{x^2+y^2}{17} + \frac{x+y}{10} = 0,\ xy = 1.$

8. A person bought a certain number of sheep for £210. He lost 10, and to make up the deficiency sold the remainder at 10*s.* profit per head. How many did he buy?

9. Prove the rule for the summation to infinity of a geometrical progression; and sum to n terms and to infinity $8\frac{1}{3} + 5 + 3 + \cdots\cdots$

10. The seventh term of an arithmetical progression is 1; and the sum of twenty-five terms is zero. Find the progression.

11. If $a : b :: c : d$, prove that $a + b : a - b :: c + d : c - d$.

12. If $2x + 3y : 2x - 3y :: 2a^2 + 3b^2 : 2a^2 - 3b^2$, then x has to y the duplicate ratio that a has to b.

XLIV.

UNIVERSITY OF OXFORD, ENG.

Local Examination, Senior Candidates, June, 1881. — *Time Allowed,* $2\frac{1}{2}$ *hours.*

No credit will be given for any answer, the full working of which is not shown.

I. ALGEBRA TO QUADRATIC EQUATIONS.

1. Prove that

$$(a+b)(a+x)(b+x) - a(b+x)^2 - b(a+x)^2 = (a-b)^2 x,$$

and divide $a^3 - b^3$ by $a^{\frac{3}{2}} - 2ab^{\frac{1}{2}} + 2a^{\frac{1}{2}}b - b^{\frac{3}{2}}$.

2. Resolve into component factors

(i.) $63x^3y - 28xy^3$; (ii.) $a^5 - a^4b - ab^4 + b^5$.

Find the remainder when $a^n + b^n$ is divided by $a - b$.

3. Find the G.C.M. of

$x^4 - 6x^3 + 13x^2 - 12x + 4$ and $x^4 - 4x^3 + 8x^2 - 16x + 16$, and the L.C.M. of $x^3 - y^3$, $x^3 + y^3$, $x^3 - xy^2$, and $x^2y + xy^2 + y^3$.

4. Simplify the fractions

(i.) $\dfrac{x+2}{x+1} - \dfrac{x+1}{x+2} - \dfrac{2}{x+4}$;

(ii.) $\dfrac{1}{1-\dfrac{1}{x}} - \dfrac{1}{\dfrac{1}{x^3}-\dfrac{1}{x}} - \dfrac{1}{1-\dfrac{2x}{x^2+1}}$;

(iii.) $\dfrac{\sqrt{x}+\sqrt{x-1}}{\sqrt{x}-\sqrt{x-1}} - \dfrac{\sqrt{x}-\sqrt{x-1}}{\sqrt{x}+\sqrt{x-1}}$.

5. Solve the equations

(i.) $\frac{1}{8}\{(2x-32)-(x+16)\} = \frac{1}{11}\{(x-20)-(2x-11)\}$;

(ii.) $(x+5)(y+7) = (x+1)(y-9) + 112$, $2x+5 = 3y-4$;

(iii.) $x - \dfrac{x^3-8}{x^2+5} = 2$.

6. A person invests £500, part of it at 5 per cent and the remainder at 3 per cent; and he thus gets $4\frac{1}{2}$ per cent on the whole. How much does he invest at each rate of interest?

7. Find the square root of $9 - 24x - 68x^2 + 112x^3 + 196x^4$.

II. Higher Algebra.

8. Prove that a ratio of greater inequality is diminished, and a ratio of less inequality is increased, by adding the same number to each of its terms.

If $a : a-b :: c : c-d$, then $a+b : b :: c+d : d$.

9. Prove that the geometrical mean between two numbers is also the geometrical mean between the arithmetical and harmonical means.

Sum the series:

(i.) $25\frac{1}{2}+24+22\frac{1}{2}+21+\cdots\cdots$ to 15 and to 21 terms;

(ii.) $4\frac{4}{15}+2\frac{2}{3}+1\frac{2}{3}+\cdots\cdots$ to n terms, and to infinity.

10. Prove that the number of combinations of n things taken r together is equal to that of n things taken $n-r$ together, and greater than that of $n-1$ things taken $r-1$ together.

How many different numbers can be made with all or any of the figures of the number 1881?

11. Employ the binomial theorem to expand $(a-x)^{11}$; also $(a+x)^{\frac{3}{2}}$ in ascending powers of x to 5 terms.

12. If α, β are the roots of the equation $ax^2+bx+c=0$, find the values of $\alpha+\beta$ and $\alpha^3+\beta^3$ in terms of a, b, c.

13. Solve the equations

(i.) $x^3-\dfrac{1}{x^3}+7(x^3+1)=0$;

(ii.) $3x^2+y^2=3xy+7=19$.

XLV.

University of Oxford, Eng.

Local Examination, Junior Candidates, June, 1882. — *Time Allowed,* $2\frac{1}{2}$ *hours.*

No credit will be given for any answer, the full working of which is not shown.

I. Algebra.

1. Find the value of $\dfrac{x^2-y^2}{x^2+y^2}$,

(i.) when $x=\dfrac{a+b}{2}$, $y=\dfrac{a-b}{2}$;

(ii.) when $x=\frac{1}{3}$, $y=-\frac{1}{4}$.

2. Square $2b-3c$, and find the product of $a+2b-3c$ and $a-2b+3c$.

3. Find the G.C.M. of $2a^5b+2a^2b^4$ and $4a^5+4a^3b^2+4ab^4$; and the L.C.M. of $5x(x^2+2x+1)$, $10x^2(x^2-1)$, and $15(x+1)(x^2-2x+1)$.

4. Simplify

(i.) $\dfrac{4a^4}{x^4-a^4}+\dfrac{2a^2}{x^2+a^2}+\dfrac{a}{a+x}+\dfrac{a}{a-x}$;

(ii.) $\left[x-\dfrac{1}{1+\dfrac{2}{x-1}}\right]\div\dfrac{x+\dfrac{1}{x}}{\dfrac{1}{x}+1}$.

5. Solve the equations

(i.) $\dfrac{1}{2}-\dfrac{5-x}{x-2}=\dfrac{5-x}{x-2}-\dfrac{1}{2}$;

(ii.) $2x+\dfrac{y}{2}=4,\ 2y+\dfrac{x}{2}=1$.

6. A and B set out at the same time from the same spot to walk to a place 6 miles distant and back again. After walking for 2 hours, A meets B coming back. Supposing B to walk twice as fast as A, and each to maintain uniform speed throughout, find their respective rates of walking.

II. Higher Algebra.

7. Solve the equations

(i.) $x^2+\left(\dfrac{1}{a}-\dfrac{1}{b}\right)x=\dfrac{1}{ab}$;

(ii.) $\dfrac{1}{2}\left[\dfrac{1}{(x+2)(2x-1)}-\dfrac{1}{(x-2)(2x+1)}\right]=\dfrac{1}{4x^2-1}$;

(iii.) $\dfrac{x+y}{x-y}=\dfrac{3}{2},\ x^2-9y^2=16$.

8. Show that the sum of any two consecutive whole numbers is equal to the difference of their squares.

9. Find the sum to n terms of an arithmetical progression, the first term and the common difference being given.

What is the amount of a debt which can be discharged in two years by the payment of 10*s.* the first month, £1 the second, 30*s.* the third, and so on, no interest being exacted?

10. Sum to n terms, and to infinity, the series $\frac{2}{3} - \frac{1}{2} + \frac{3}{8} - \cdots\cdots$

11. If $a_1 : b_1 :: a_2 : b_2$, prove that

$$(a_1 + a_2)^2 : (b_1 + b_2)^2 :: a_1^2 + a_2^2 : b_1^2 + b_2^2.$$

12. Show that if b is a mean proportional between a and c, then $(a^2 + b^2)(b^2 + c^2) = (ab + bc)^2$.

XLVI.

University of Oxford, Eng.

Local Examination, Senior Candidates, June, 1882.— *Time Allowed,* $2\frac{1}{2}$ *hours.*

No credit will be given for any answer, the full working of which is not shown.

Candidates are reminded that in order to pass in mathematics, they must satisfy the examiners in the first part of this paper.

I. Algebra to Quadratic Equations.

1. Prove that $(a + 2b)^3 = a^3 + 2b^3 + 6b(a + b)^2$, and find the quotient $a^2 + ab^{\frac{1}{2}} + b \div a + a^{\frac{1}{2}}b^{\frac{1}{4}} + b^{\frac{1}{2}}$.

2. Simplify

(i.) $\dfrac{x-y}{x+y}[(x+2y)^3 + (2x+y)^3]$;

(ii.) $\dfrac{1}{1+\dfrac{x}{y+z}} + \dfrac{1}{1+\dfrac{y}{z+x}} + \dfrac{1}{1+\dfrac{z}{x+y}}$;

(iii.) $\dfrac{(b-c)^2}{(c-a)(a-b)} + \dfrac{(c-a)^2}{(a-b)(b-c)} + \dfrac{(a-b)^2}{(b-c)(c-a)}$.

3. Find the G.C.M. of $3x^3 + 17x^2 + 22x + 8$ and $6x^3 + 25x^2 + 23x + 6$, and the L.C.M. of $(x^4 - y^4)^2$, $x^3 + y^3$, $x^3 - y^3$, and $(x^2 - y^2)^2$.

4. Extract the square root of
   (i.) $7 - 4\sqrt{3}$;
   (ii.) $x^3(x^3 + 2) + 2x^2(x^2 + 1) - x(x - 2) + 1$.

5. Solve the equations
   (i.) $11x - 5(x - 6) - 6(3x - 11) + 9(x - 7) = 0$;
   (ii.) $\frac{x}{8} + \frac{y}{6} = \frac{x}{12} + \frac{y}{4} = 20$;
   (iii.) $(3x - 6)^2 + 7x^2 - 256 = 0$.

6. My income of £240 is derived from two sums invested at three and nine per cent respectively; but, if the rates of interest were interchanged, my income would be doubled; find the sums invested.

II. Higher Algebra.

7. If $a : b :: c : d$, prove that
   (i.) $a^3 + d^3 > b^3 + c^3$;
   (ii.) $3a + 2b : 3a - 2b :: 3c + 2d : 3c - 2d$.

8. Find an expression for the sum of n terms of a geometrical series, and explain the expression "sum to infinity."

   Find the fourth term in
   (i.) $2 + 2\frac{1}{2} + 3 + \cdots\cdots$
   (ii.) $2 + 2\frac{1}{2} + 3\frac{1}{8} + \cdots\cdots$
   (iii.) $2 + 2\frac{1}{2} + 3\frac{1}{3} + \cdots\cdots$

9. Find the number of permutations ${}^nP_r$ of n things taken r together. If ${}^nP_4 : {}^{n-1}P_5 :: 3 : 22$, find n.

10. Show that the number of terms in the expansion of $(a + x)^n$ is $n + 1$, if n is a positive integer.

    Apply the binomial theorem to find $(10.001)^7$ to five places of decimals.

11. If α, β are the roots of $ax^2 + bx + c = 0$, find the equation whose roots are
(i.) α^2, β^2; (ii.) $\alpha(1+\beta), \beta(1+\alpha)$.

12. Solve the equations
(i.) $144x^2 - 1 + 6\sqrt{9x^2 - x} = 16x$;
(ii.) $x^2(x^2 - y^2) = 25,\ y^2(x^2 + y^2) = 19\frac{1}{8}$;
(iii.) $\sqrt[3]{14 - x} + \sqrt[3]{14 + x} = 4$.

XLVII.

University of Oxford, Eng.

First Examination of Women, May, 1880. — *Time Allowed,* 2½ *hours.*

1. Find the value of $\dfrac{b^2c - 2a^2b}{a^3 + 4b^3}$ when $a = 1$, $b = -\frac{1}{2}$, $c = 0$.
2. Take $a + 2b + 3c - 4d - 5e$ from $3a - 4b + c - d + e$.
3. Multiply $a^3 - a^2b + ab^2 - b^3$ by $a + b$,
and divide $a^4 + a^2b^2 + b^4$ by $a^2 + ab + b^2$.
4. Find the H.C.F. of $a^4 + 5a^2 - 6$ and $a^4 + 5a^3 + 4$, and the L.C.M. of $12(a^3 - b^3)$, $15(a^3 + b^3)$, $20ab(a^2 - b^2)$.
5. Simplify $\dfrac{a^2 + 4a + 3}{a^2 - a - 2}$.
6. Extract the square root of $a^4 - 2a^3 + 2a^2 - a + \frac{1}{4}$.
7. Solve
(i.) $7(x-1) - 6(x-2) = 3(x-3)$;
(ii.) $\dfrac{x}{9} + \dfrac{x}{6} + \dfrac{x}{3} = x - 7$;
(iii.) $(x-3)(x-13) = (x-4)(x-9)$;
(iv.) $8x + 3y = 74,\ 9x - 2y = 51$.
8. I have in my purse £1 13*s.* 9*d.* made up of a certain number of pence, twice the same number of farthings, and thrice the same number of fourpenny pieces. Find the number of each coin.

9. A is thrice as old as B. Seven years ago A was four times as old as B. Find their ages now.

10. A and B play at cards. A wins six shillings, and finds he has thrice as much as B. The game is continued till A finds he has lost twenty-four shillings, and then has a third of what B has. With what sum did each begin?

XLVIII.

University of Oxford, Eng.

Second Examination of Women, May, 1880. — *Time Allowed*, 2 *hours.*

1. Divide $a^{\frac{5}{2}} - a^{-\frac{5}{2}}$ by $a^{\frac{1}{2}} - a^{-\frac{1}{2}}$.

2. Simplify

$$\frac{(6a^2 - a - 2)(8a^2 - 10a + 3)(12a^2 + 17a + 6)}{(8a^2 - 2a - 3)(12a^2 + a - 6)(6a^2 + a - 2)};$$

$$\frac{6a^2 - 17a + 12}{12a^2 - 25a + 12} + \frac{27a^2 + 18a - 24}{12a^2 + 7a - 12} + \frac{25a^2 - 25a + 6}{20a^2 - 23a + 6}.$$

3. Extract the square root of

(i.) $4a^2 + \dfrac{9}{a^2} - 11 - 6a^{-1} + 4a$;

(ii.) $41 - 12\sqrt{5}$.

4. Solve the equations

(i.) $2\dfrac{x+a}{x+b} + 3\dfrac{x+b}{x+a} = 5$;

(ii.) $x^2 - 7x + 10 = 0$;

(iii.) $\sqrt{x-2} + \sqrt{x+3} = \sqrt{4x+1}$;

(iv.) $x + y = 8, \ xy = 15$.

5. If $a : b :: c : d$, prove that $a + b : a - b :: c + d : c - d$.

6. £21 18 *s.* is divided equally among a certain number of persons. If each received a penny less, each would have had as many pence as there were persons. Find the number of persons.

7. A steamer takes 2 hours 24 minutes less time to travel from A to B than from B to A. The steamer travels at the rate of 16 miles an hour, and the stream flows at the rate of 6 miles an hour. Find the distance of A from B.

XLIX.

University of Oxford, Eng.

First Examination of Women, June, 1882. — *Time Allowed,* $2\frac{1}{2}$ *hours.*

1. Find the value of $x(y+z)+y[x-(y+z)]-z[y-x(z-x)]$ when $x=3$, $y=2$, $z=1$.

2. Subtract $2x^4+\frac{5}{3}x^2-x+\frac{1}{4}$ from $\frac{5}{2}x^4+x^2-\frac{1}{4}x-\frac{3}{4}$.

3. Multiply $1+2x+3y+4x^2-6xy+9y^2$ by $1-2x-3y$.

4. Divide $x^4-\frac{13}{6}x^3+x^2+\frac{4}{3}x-2$ by $x-\frac{3}{2}$.

5. Resolve into factors
   (i.) $4x^4-36x^2y^2$;
   (ii.) $(2x-3y)^2-(x-2y)^2$.

6. Find the G.C.M. of x^3-3x+2 and x^3+4x^2-5.

7. Find the L.C.M. of
   $12(1-x^2)$, $15(1-x)^2$, and $20(x+x^2)$.

8. Simplify
   (i.) $\frac{8a^2b}{c}\times\frac{c^2d}{8a^3}\times\frac{4ab}{cd}\times\frac{bcd-cd^2}{4b^2-4bd}$;

   (ii.) $\left(\frac{1}{1+4x}+\frac{4x}{1-4x}\right)\div\left(\frac{1}{1-4x}-\frac{4x}{1+4x}\right)$.

9. Solve the equations

(i.) $\frac{x}{3}+\frac{2x}{9}-\frac{x}{27}-\frac{x+1}{11}=23;$

(ii.) $\frac{x+a}{x+b}+\frac{x+b}{x+c}=2;$

(iii.) $\frac{2x-3}{y}=1\frac{3}{4},\quad \frac{2y+3}{x}=2\frac{1}{5}.$

10. A person walks a certain distance at the rate of $3\frac{1}{2}$ miles an hour, and finds that if he had walked 4 miles an hour, he would have gone the same distance in less time by one hour; what is the distance?

11. Find two numbers such that, if half the first be added to the second, or $\frac{1}{3}$ of the second be added to the first, the sum will in either case be 30.

12. Find the square root of

$a^6-4a^5+8a^4-10a^3+8a^2-4a+1.$

L.

University of Oxford, Eng.

Second Examination of Women, June, 1882. — *Time Allowed,* 2 *hours.*

1. Find the value of $(a+b-c)^2+(b+c-a)^2+(c+a-b)^2$ when $a=2$, $b=3$, $c=-\frac{1}{2}$.

2. If $a+b+c=p$, $bc+ca+ab=q^2$, and $abc=r^3$, prove that $a^3+b^3+c^3=p^3-3pq^2+3r^3$.

3. Find the G.C.M. of

$2x^3+x^2-12x+9$ and $2x^3-7x^2+12x-9$, and simplify $\frac{1}{2x^3-7x^2+12x-9}-\frac{1}{2x^3+x^2-12x+9}.$

4. Add together $\frac{x-a}{(x+a)^2}$, $\frac{x+a}{(x-a)^2}$, and $\frac{2x(3a-x)}{(x-a)(x+a)^2}.$

5. Find the square root of
$$x^6 + a^2x^4 + 2a^3x^3 + \tfrac{1}{4}a^4x^2 - a^5x + a^6.$$

6. Solve the equations

(i.) $\dfrac{1}{2x-1} + \dfrac{2}{4x-3} = \dfrac{1}{x-1}$;

(ii.) $\sqrt{5x^2 + x - 16} = 3x - 2$;

(iii.) $(x-y)(x-3y) = 24,\ x - 2y = 5.$

7. There are 250 flowers in a conservatory; the number of geraniums is five times the number of roses, and is less by 30 than the number of other flowers. How many roses, geraniums, and other flowers respectively are there in the conservatory?

8. Find the value of x in each of the following proportions:

(i.) $1 : x :: 2 : 3$; (ii.) $1 : 2 :: 3 : x$; (iii.) $4 : x :: x : 9$.

9. Two persons, A and B, start at the same time by their watches, from two places 24 miles apart, and drive towards each other at rates which are as $2 : 3$; but in consequence of A's watch being $\frac{1}{4}$ hour too fast, and B's $\frac{1}{4}$ hour too slow, they meet half way. At what rate does each drive?

LI.

University of Oxford, Eng.

First Examination of Women, June, 1883. — *Time Allowed,* $2\frac{1}{2}$ *hours.*

1. Evaluate $(x-y)^2 + (y-z)^2 + (z-x)^2$, when $x = 3\frac{1}{2}$, $y = 2\frac{1}{2}$, $z = 1\frac{1}{2}$.

2. From the sum of $\frac{1}{6}(2x - 3y + 4z)$ and $\frac{1}{12}(4x + 3y - 11z)$ subtract $\frac{1}{24}(8x - 9y + 6z)$.

3. Multiply $x^2 + y^2 + a(x - y)$ by $xy - a(x + y) + a^2$.

4. Divide $x^3 + 8y^3 - 125z^3 + 30xyz$ by $x + 2y - 5z$.

5. Express in factors
   (i.) $7x^2 - 77x - 182$;
   (ii.) $20x^4 - 60x^3y + 45x^2y^2$.

6. Find the G.C.M. of $x^2 + 11x + 30$, $9x^3 + 53x^2 - 9x - 18$.

7. Find the L.C.M. of
   $15x^2(a^2 - 2ax + x^2)$, $21a^2(a^2 + 2ax + x^2)$, $35ax(a^2 - x^2)$.

8. Simplify $\dfrac{\dfrac{a}{a-b} + \dfrac{b}{a+b} - 1}{\dfrac{a}{a-b} - \dfrac{b}{a-b} + 1}$, and find the value of
   $\dfrac{x+2}{2-x} + \dfrac{x-2}{2+x} - \dfrac{4}{4-x^2}$ when $x = \frac{1}{2}$.

9. Solve the equations
   (i.) $\frac{2}{3}(x-1) + 3\left(\dfrac{x}{2} - 9\right) - (x - 13) = 11$;
   (ii.) $\dfrac{1}{x-a} + \dfrac{1}{x-b} - \dfrac{2}{x-a-b} = 0$;
   (iii.) $\dfrac{5x+7y}{7} = \dfrac{7x+5y}{8} = 8$.

10. The sum of three consecutive whole numbers exceeds the greatest of them by 19; what are the numbers?

11. Find the fraction which is such that if 3 be subtracted from the numerator, and 5 added to the denominator, the value is $\frac{1}{4}$; but if 5 be subtracted from the numerator, and 3 added to the denominator, the value is $\frac{1}{6}$.

12. Extract the square root of
    $36x^2 - 120ax - 12a^2x + 100a^2 + 20a^3 + a^4$.

LII.

University of Oxford, Eng.

Second Examination of Women, June, 1883.— *Time Allowed,* 2 *hours.*

1. Divide $x^5 - y^5 - xy(x^3 + 2y^3)$ by $(x+y)(x-y) - xy$, and verify the result when $x = 2y = \frac{1}{2}$.

2. Prove that $(cy - bz)^2 + (az - cx)^2 + (bx - ay)^2 = x^2 + y^2 + z^2 - (ax + by + cz)^2$ if $a^2 + b^2 + c^2 = 1$.

3. Find the G.C.M. and the L.C.M. of
$7x^3 - 2x^2 - 5$ and $7x^3 + 12x^2 + 10x + 5$.

4. Simplify

(i.) $\dfrac{y(x^3 - y^3)}{x(x+y)} \times \dfrac{(x^2 - y^2)^2}{(x^2x^2 + xy + y^2)} \div \dfrac{(x-y)^3}{(x+y)^2}$;

(ii.) $\dfrac{a}{b^2\left(\dfrac{1}{a} + \dfrac{1}{b}\right)} + \dfrac{b}{a^2\left(\dfrac{1}{a} - \dfrac{1}{b}\right)} + \dfrac{2ab}{a^2 - b^2}$.

5. Find the square root of

$$(x^2 + y^2)(x^2 + z^2) + 2x(x^2 + yz)(y + z) + 4x^2yz.$$

6. Solve the equations

(i.) $\dfrac{x-3}{2\frac{1}{2}} - \dfrac{2x-1}{3\frac{3}{4}} + \dfrac{7x-11}{5\frac{5}{8}} = 6$;

(ii.) $\dfrac{5(3x^2 - 1)}{1 + 5x} + \dfrac{2}{x} = 3x$;

(iii.) $x^2 + y^2 = 9, \quad x - 3y + 3 = 0$.

7. A number of men are employed at 13*s.* 6*d.* per week each, and women at 10*s.* 6*d.*; the number of women exceeds that of the men by 6, but the men's wages amount to 27*s.* per week more than the women's; how many of each sex are employed?

8. Find the mean proportional between $\frac{15}{2}$ and $\frac{5}{6}$; and the fourth proportional to 9, 11, 27.

   If the first two terms of a proportion are 3 and 2, and the third term exceeds the fourth by 5, what are the third and fourth terms?

9. Divide £500 among three persons, so that the share of the first may be to that of the second as $8:17$, and the share of the third to the other two together as $3:2$.

LIII.

University of London, Eng.

Matriculation Examination, Arithmetic and Algebra, June, 1877.

1. Divide $\frac{25}{21}+6\frac{3}{8}-7\frac{1}{28}$ by $5\frac{2}{3}-3\frac{1}{7}+\frac{17}{12}$, and multiply the result by $6\frac{5}{13}+9\frac{7}{25}-13\frac{1}{5}$.

2. Find what fraction of a guinea is equal to the difference between $\frac{2}{7}$ of a crown and $\frac{13}{11}$ of a shilling.

3. Calculate to five places of decimals the fraction
$$\frac{3.70271\times 0.64732}{0.043679}.$$

4. Reduce the circulating decimal $1.5\dot{2}37\dot{2}$ to a vulgar fraction in its lowest terms.

5. Extract the square roots, to five places of decimals, of the numbers, 3.9726523, 0.39726523, and 0.039726523.

6. Simplify the algebraic expression,
$$\frac{b}{3ax-5by}-\frac{1}{\dfrac{ax}{b}-y-\dfrac{2by^2}{3ax-2by}}.$$

7. Divide $15x^5-17x^4-24x^3+138x^2-130x+63$ by $5x^3+6x^2-9x+7$, and verify the result.

8. Prove that if $\dfrac{x^2-xy+y^2}{u^2-uv+v^2}=\dfrac{x^3+y^3}{u^3+v^3}\times\dfrac{v}{y}$, then $\dfrac{x}{y}=\dfrac{u}{v}$.

9. Given the first term, the middle term, and the number of terms in an arithmetical progression; find the sum of the series. Has this problem a meaning, if the number of terms is even?

10. Find to n terms and to infinity the sum of the geometrical progression in which the fourth term is 1 and the ninth term is $\frac{1}{243}$.

11. Solve the equations

(i.) $\dfrac{1-3x}{2}+\dfrac{3x+1}{2}=\dfrac{2}{1-3x}$;

(ii.) $2x-4y=7,\ 3x+7y=19.$

12. A tourist, having remained behind his companions, wishes to rejoin them on the following day. He knows that they are 5 miles ahead, and that they will start in the morning at eight o'clock, and will walk at the rate of $3\frac{1}{4}$ miles an hour. When must he start, in order to overtake them at one o'clock, P.M., walking at the rate of 4 miles an hour, and resting once for half an hour on the road?

LIV.

Science Schools and Classes, England.

Mathematics, First Stage, May, 1880.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Find the value, when $x=5$ and $y=3$, of

$$\frac{x^4-4x^3y+6x^2y^2-5xy^3+2y^4}{2x^4-5x^3y+6x^2y^2-4xy^3+y^4}. \qquad [6.]$$

2. Multiply a^3-x^3 by a^2-x^2, and divide the product by $(a-x)^2$. [8.]

3. Simplify $(a-b)(b+c)(c+a)+(b-c)(c+a)(a+b)+(c-a)(a+b)(b+c)$, and find its value when $a=1$, $b=3$, and $c=-2$. [10.]

4. Solve the equations [12.]

(i.) $7\left(x+\frac{1}{3}\right)-5x\left(\frac{1}{3x}+\frac{1}{2\frac{1}{2}}\right)=4.$ [6.]

(ii.) $0.5x+0.07y=0.93,\ 0.03x-0.4y=0.46.$ [6.]

5. The rent of a shop is $\frac{2}{7}$ of the rent of the whole house of which it is a part. Being separately rated, its occupier pays £10 15*s.* 0*d.* a year less in rates than the occupier of the rest of the house. The rates are 3*s.* 7*d.* in the pound. What is the rent of the whole house? [14.]

6. Find, as a fraction in its lowest terms, the value of

$$\frac{1}{x^3-3x^2-15x+25}-\frac{1}{x^3+7x^2+5x-25}.$$ [12.]

LV.

Science Schools and Classes, England.

Mathematics, Second Stage, May, 1880. — *Arithmetic and Algebra.*

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Assuming that a franc is worth 9.504*d.*, and a hard dollar 50.49*d.*, what is the smallest sum in francs that can be exactly paid in hard dollars? [16.]

2. Show how to find the square root of a vulgar fraction, so as to make sure of obtaining it in a finite form, if it has one.

Ascertain whether the square roots of the following fractions are finite or not: $\frac{343}{1183}$, $\frac{14641}{243}$, $\frac{329}{81}$, $\frac{99}{1331}$. [18.]

3. Solve two of the following sets of equations: [20.]

(i.) $\sqrt{x^2+a^2}-\frac{b^2}{\sqrt{x^2+a^2}}=\frac{b^2-a^2}{a}.$ [10.]

(ii.) $x+y=7,\ x^3+y^3=133.$ [10.]

(iii.) $xy+\sqrt{x+y}=11,\ 2xy-\sqrt{x+y}=13.$ [10.]

4. What is the least integral multiplier which will make $17x^5-68x^4y+102x^3y^2-68x^2y^3+17xy^4$ a complete cube? [20.]

5. A rectangular plot of ground measures 42 acres, and its diagonal is 1243 yards long. What are its sides? [22.]

6. Two boys start at the same instant from the same corner of a square, the length of one of whose sides is 200 yards, and they run round it in opposite directions: one (A) runs at the rate of 100 yards in 15 seconds, and loses 2 seconds in turning a corner; the other (B) runs at the rate of 100 yards in 16 seconds, and loses 1 second in turning a corner. Where do they meet? [26.]

LVI.

Science Schools and Classes, Eng.

Mathematics, First Stage, May, 1881.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Show that

$$\frac{x^2}{a^2}+\left(\frac{z-x}{b}\right)^2 \text{ and } \frac{z^2}{a^2+b^2}+\frac{a^2+b^2}{a^2b^2}\left(x-\frac{za^2}{a^2+b^2}\right)^2$$

are identical expressions; that is to say, that the one can be deduced from the other. Find their values, when $x=3$, $z=4$, and $a=b=5$. [8.]

2. Simplify $\left(\frac{bc}{a}\right)^{m+n}\left(\frac{a}{c}\right)^m\left(\frac{a}{}\right)^n$, and find its value when $a=7$, $b=3$, $c=2$, $m=2$, $n=1$. [8.]

3. Find the G.C.M. and L.C.M. of $2x^4+9x^2+5x+12$ and $2x^4+4x^3+13x^2+11x+12$. [12.]

4. Solve the equations [12.]

(i.) $(2+3x)\left(4-\frac{5}{x}\right)=(3-2x)\left(6-\frac{7}{x}\right)$; [6.]

(ii.) $(x+y)^2-(x-y)^2=352$, $x(y+5)=143$. [6.]

5. Of two squares of carpet, one measures 44 feet more round than the other, and 187 square feet more in area. What are their sizes? [12.]

6. Oranges are bought for half-a-crown a hundred; some are sold at $3s.\ 6d.$ a hundred, and the rest at $2s.\ 10\frac{1}{2}d.$ a hundred. The same profit is made, as if they had all been sold at $3s.\ 1\frac{1}{2}d.$ a hundred. Of a thousand oranges sold, how many fetch $3s.\ 6d.$ a hundred? [14.]

LVII.

Science Schools and Classes, Eng.

Mathematics, First Stage, May, 1882.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Divide $x^3+8y^3-27z^3+18xyz$ by $x+2y-3z$, and test your answer by substituting $x=5$, $y=-4$, $z=3$ in the dividend, divisor, and quotient. [12.]

2. Show that the product of $1-\frac{a}{x+2a}$, $1+\frac{b}{x-2b}$, and

$$x+2(a-b)-\frac{4ab}{x} \text{ is } \frac{(x+a)(x-b)}{x}.$$

Write down the value of each factor and of the product, when $x=3a=3b$. [12.]

3. Reduce to its lowest terms $\frac{x^4-13x^2+36}{x^4-x^3-7x^2+x+6}$. [12.]

4. Divide x^2+px+q by $x-a$, and find the relation that must hold good between a, p, and q, when the division can be performed without leaving a remainder. [12.]

5. Solve the equations [12.]

(i.) $\frac{1}{3}(0.75-x)+\frac{1}{5}(0.47+2x)=(3-\frac{1}{15})x$. [6.]

(ii.) $\frac{x}{15}+\frac{y}{12}=\frac{x}{3}-\frac{y}{4}=1$. [6.]

6. A man has 1000 apples for sale; at first he sells so as to gain at the rate of 50 per cent on the cost price; when he has done this for a time the sale falls off, so he sells the remainder for what he can get, and finds that by doing so he loses at the rate of 10 per cent; if his total gain is at the rate of 29 per cent, how many apples did he sell for what he could get? [14.]

LVIII.

Science Schools and Classes, England.

Mathematics, Second Stage, May, 1882.—Arithmetic and Algebra.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Given $\sqrt{5}=2.236068$, express $\frac{1}{\sqrt{20}}$ and $\frac{\sqrt{5}-2}{\sqrt{5}+2}$ as decimals, true to the fifth place. [18.]

2. Solve the equations [25.]

(i.) $x^3-2x^2-3x+4=0$;

(ii.) $x^2+y=8,\ 3x+2y=7$.

3. A farmer sold 7 oxen and 12 cows for £250. He sold 3 more oxen for £50 than he did cows for £30. Required the price of each. [25.]

4. What is the term which must be added to $9x^4+12x^3+20x+25$ to make it a complete square? [25.]

5. Find $(x+2y)^7$, and obtain the sixth root of 6,321,363,049. [25.]

6. It is known that the volume of a cylinder varies as the base and height jointly. If the volume of the first of two cylinders is to that of the second as 11 : 8, and the height of the first is to that of the second as 3 : 4, and if the base of the first has an area of 16.5 square feet, what is the area of the base of the second? [25.]

LIX.

Science Schools and Classes, England.

Mathematics, First Stage, May, 1883.

Not more than three questions are to be answered. The number of marks assigned to each question is given in brackets.

1. Explain why the product is a^{12}, when a^5 is multiplied by a^7, and why the quotient is a^5 when a^8 is divided by a^3. [8.]

2. Obtain $(x^3+3x+15)^2-(x^3-3x+15)^2$ in its simplest form, and find its value when $2x=-5$. [12.]

3. Simplify the expressions [12.]

(i.) $\dfrac{x-a}{\dfrac{1}{a}-\dfrac{1}{b}} \times \dfrac{a-b}{1-\dfrac{a}{x}}$;

(ii.) $\left(\dfrac{1}{2}+\dfrac{1}{3x}\right) \div \left(9x-\dfrac{4}{x}\right)$.

4. Find the G.C.M. of $x^4-5x^3-6x^2+35x-7$ and $3x^3-23x^2+43x-8$, and write down these expressions in factors. [12.]

5. Solve the equations [12.]

(i.) $x - \frac{2x - 0.3}{0.7} = \frac{5 - x}{0.35}$;

(ii.) $\frac{3y}{4} - \frac{2x}{5} = \frac{13}{5}$, $\frac{x}{4} + \frac{y}{5} = \frac{19}{12}$.

6. A sum of £23 14*s.* is to be divided between A, B, C; if B gets 20 per cent more than A and 25 per cent more than C, how much does each get? [16.]

LX.

Civil Service of Great Britain.

Competitive Examination of Candidates for Inspectorships of National Schools, Ireland, 1878. — *Time Allowed,* 3 *hours.*

1. Find the value of $\frac{a^3 + a^2 - a - 1}{b^3 - b^2 + b - a}$ when $a = 2$, $b = \frac{1}{2}$, and the value of $(1 + a)^{\frac{1}{2}} \times (1 - b)^{-\frac{1}{2}}$ when $a = \frac{1}{5}$, $b = \frac{1}{6}$.

2. Divide $6x^3 + x^2 + x + 2$ by $2x^2 - x + 1$.
Find the first four terms in the quotient obtained when $1 + x$ is divided by $1 - 2x$.

3. Simplify the expressions

(i.) $(16a^6b^2)^{\frac{1}{2}} \times (a^{\frac{1}{6}}b^{\frac{1}{2}})^2 \times (2a^{\frac{2}{3}}b)^{\frac{1}{2}}$;

(ii.) $(1+a+b)^3 + (1+a-b)^3 + (1-a+b)^3 + (1-a-b)^3$.

4. Multiply $a^3b^{-3} - ab^{-1} + a^{-1}b - a^{-3}b^3$ by $ab^{-1} + a^{-1}b$, and divide $(x^2y^{mn})^{\frac{1}{m}}$ by $(x^{\frac{1}{m}}y^2)^{mn}$.

5. Reduce the fraction $\frac{3x^3 - x^2 - x - 1}{3x^3 - 4x^2 - x + 2}$ to its lowest terms.

6. Find the L.C.M. of

(i.) $7a^3bx^2$, $3ax^3$, $6ab^3$;

(ii.) $3x^2 - x - 2$, $2x^2 + x - 3$.

7. Find the value of a which will make $x^4 - x^3 - x^2 - ax$ divisible, without remainder, by $x^2 + x$.

8. Solve the equations

(i.) $1 - \frac{2}{x} - \frac{3}{2x} = \frac{4}{3x}$; (ii.) $\frac{x}{1-x} + \frac{1-2x}{2-x} = 1$.

9. Extract the square roots of

(i.) $9 - 4\sqrt{2}$; (ii.) $x^2 - 6x + 13 - 12x^{-1} + 4x^{-2}$.

10. Solve the equations

(i.) $\sqrt{x} - \sqrt{1+x} = \frac{1}{\sqrt{x}}$;

(ii.) $\frac{1-ax}{1+ax} \times \frac{a-x}{a+x} = 1$;

(iii.) $x^2 + xy = 32,\ y^2 - xy = 18$.

11. A train is timed to travel between two stations, A and C, at 45 miles an hour. It travels from A to an intermediate station, B, at the rate of 40 miles per hour, and the speed is then increased to 50 miles an hour. The train arrives punctually at the time appointed. Compare distances of B and C from A.

LXI.

Civil Service of Great Britain.

Open Competitive Examination for Clerkships of the Superior Class in the India Office, 1879.— *Time Allowed,* 3 *hours.*

Full marks may be obtained by doing less than the whole of this paper.

1. If m and n be positive integers, prove that $a^m \times a^n = a^{m+n}$; and, assuming this formula to hold good for all values of the indices, deduce the meanings of $a^{\frac{m}{n}}$, a^0, and a^{-n}.

2. Divide

$x^4+y^4+z^4-2(x^2y^2+y^2z^2+z^2x^2)$ by $x^2+2yz-y^2-z^2$,

and multiply

$2x^{\frac{2}{3}}y^{\frac{1}{3}}+7x^{\frac{1}{3}}y^{\frac{2}{3}}+12y$ by $x^{\frac{2}{3}}y^{-1}-x^{\frac{1}{3}}y^{-\frac{2}{3}}+x^{\frac{1}{3}}y^{-\frac{1}{3}}$.

3. Prove that the L.C.M. of any two algebraical expressions is equal to their product divided by their G.C.M.

Find the G.C.M. and L.C.M. of the expressions

$(x^2+b^2)c+(b^2+c^2)x$ and $(x^2-b^2)c+(b^2-c^2)x$.

4. Simplify the following expressions:

(i.) $$\frac{1+\dfrac{4x^2}{6xy+9y^2}}{1+\dfrac{9y^2}{4x^2-6xy}}\div\left(\frac{16x^4}{81y^4}-\frac{2x}{3y}\right);$$

(ii.) $$\left(\frac{a+y}{a^2+2ax-ay-2xy}-\frac{a+2x}{a^2+ay-2ax-2xy}\right)\times\frac{a^2+2ax+ay+2xy}{3a^2+ay+2ax-2xy};$$

(iii.) $$\frac{2\sqrt{a-b}}{3\sqrt{a+b}-2\sqrt{a-b}}+\frac{3\sqrt{a+b}}{3\sqrt{a+b}+2\sqrt{a-b}}.$$

5. Find the condition that the roots of the equation $ax^2+bx+c=0$ may be real, and show that the roots of the equation $(x+p)(x+q)=pqx^2$ will always be real if p and q are real.

6. Solve the equations

(i.) $$\frac{x+2a}{(x-2a)^2}-\frac{3}{x}=0;$$

(ii.) $$6\left(x^2+\frac{1}{x^2}\right)+5\left(x+\frac{1}{x}\right)=38;$$

(iii.) $x^3y-x^4=27,\ xy^3-x^3y=84.$

7. A man, who drives twice as fast as he can walk, finds that it takes him 9 hours to drive to a certain town and to walk back, and that when he has accomplished half of the return journey, he meets a man who set out to walk from the same place an hour and a half later, and is travelling a mile and a half per hour more slowly than himself. Find the distance of the town from the starting point, and the rate at which each man walks.

8. Three lamps of equal brilliancy are placed in three different corners of a square room. Compare the intensities of light at the fourth corner and at the centre of the room, assuming that the illumination from a source of light varies inversely as the square of its distance.

9. Show how to insert any number of harmonic means between two given quantities.

   If $2p$ and $3q$ be the pth and qth terms, respectively, of an harmonic series, prove that the $(p+q)$th term will be $6(p-q)$.

10. Prove that the number of permutations of n things taken r together is $n(n-1)(n-2)\cdots\cdots(n-r+1)$.

    In how many ways can 24 ships belonging to 4 different nations be arranged in 4 lines, each consisting of 6 ships of the same nationality?

11. Expand $(a^2-x^2)^{-\frac{3}{2}}$ to five terms, and show that the middle term in the expansion of $\left(x+\dfrac{1}{2x}\right)^{2n}$ is equal to $\dfrac{1\times3\times5\cdots\cdots(2n-1)}{1\times2\times3\cdots\cdots n}$.

12. Express the fourth root of $89-28\sqrt{10}$ as the difference of two surds, and extract the fifth root of 99,999 correct to 15 places of decimals.

LXII.

Civil Service of Great Britain.

Open Competitive Examination for Admission to the Royal Indian Engineering College, July, 1879.—Time Allowed, 3 hours.

1. Find the value of $\dfrac{a+b}{b-c}+\dfrac{b+c}{c-d}+\dfrac{c+d}{d-a}+\dfrac{d+a}{a-b}$, when $a=6$, $b=4$, $c=3$, $d=1$; and of x^3-4x^2+6x-4, when $x=1+\sqrt{-1}$.

2. Multiply $x-(2+\sqrt{3})y+(1+\sqrt{3})z$ by $x-(2-\sqrt{3})y+(1-\sqrt{3})z$; and divide $x^2-4xy+y^2$ by $x-(2+\sqrt{3})y$.

3. Explain the law of indices in the multiplication and division of algebraic quantities.

Multiply $x^{\frac{5}{2}}+x^{\frac{3}{2}}+x^{\frac{1}{2}}$ by $x^{\frac{1}{2}}-x^{-\frac{1}{2}}$; and reduce to its simplest form $(x^{\frac{1}{3}}\times x^{-\frac{3}{4}})^{-\frac{3}{5}}\div x^{-\frac{1}{6}}$.

4. Simplify the following expressions:

(i.) $\dfrac{25x^4+5x^3-x-1}{20x^4+x^2-1}$;

(ii.) $$\frac{\dfrac{a+x}{a-x}+\dfrac{a-x}{a+x}}{1-\dfrac{a^2-x^2}{a^2+x^2}}\div\frac{\dfrac{a^2+x^2}{a^2-x^2}-\dfrac{a^2-x^2}{a^2+x^2}}{1-\dfrac{a^4+x^4}{(a^2+x^2)^2}}.$$

5. Solve the equations

(i.) $\sqrt{x}+\sqrt{4+x}=\dfrac{4}{\sqrt{x}}$;

(ii.) $\dfrac{x}{10}-\dfrac{1}{5}\left(\dfrac{x+1}{x+2}\right)+\dfrac{3-2x}{20}=0$;

(iii.) $\dfrac{x+1}{x-1}-\dfrac{1}{2}\left(\dfrac{x-1}{x+1}\right)=1\frac{3}{4}$;

(iv.) $x^2+y^2=8,\ 2xy-y^2=4.$

6. A number having two digits is to the number formed by inverting the order of the digits as 8 to 3, and the sum of the two numbers is 99. Find them.

7. Prove the rule for finding the G.C.M. of two algebraical quantities.

Find the L.C.M. of

a^2-ab+b^2, a^2+ab+b^2, a^3-b^3, a^3+b^3, and $(a^2-b^2)^3$.

8. Find the square roots of

$81x^4+108x^3-24x+4$ and $327-87\sqrt{15}$.

9. If p, q, and r are the arithmetic, geometric, and harmonic means of two algebraical quantities, show that

(i.) $pr=q^2$; (ii.) $p-\frac{r}{2}=\sqrt{p^2-q^2}$.

10. Sum to n terms, and where possible to infinity,

(i.) $3,\ 2\frac{1}{3},\ 1\frac{2}{3}\ \cdots\cdots$

(ii.) $3,\ 2\frac{1}{3},\ 1\frac{22}{27}\ \cdots\cdots$

(iii.) $\frac{1}{2\times 3},\ \frac{1}{3\times 4},\ \frac{1}{4\times 5}\ \cdots\cdots$

11. Distinguish between permutations and combinations, and find the number of the latter that can be formed from n things taken r together.

In how many ways can a guard of 8 soldiers be selected from a company of 25, and in how many of them will two particular men be on guard together?

12. Find the sixth term of $\left(3x-\frac{y}{3}\right)^{-\frac{2}{3}}$, and the two middle terms of $(a-x)^9$.

13. If $\frac{a+bx}{b+cy}=\frac{b+cx}{c+ay}=\frac{c+ax}{a+by}$, show that $a^3+b^3+c^3-3abc=0$.